

# DOMINO EFFECT MOTION INVESTIGATION: A NUMERICAL APPROACH

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## Abstract

This paper presents an investigation on the motion of a falling row of dominoes with different dimensions. Motion is described in means of Falling and Collision. Dominoes are assumed not to slip on the surface. The equations are solved numerically and a comprehensive simulation program is developed by the means of angle specification of dominoes as a function of time. The program works for any given arrangement of dominoes with different heights and distances. Video processing is used to measure the angle of dominoes in real experiments. Precise agreement between experimental results and simulation verifies the theory. In each collision, some percentage of the energy remains, some is transferred and the other part is wasted. These percentages are compared for different height increase rates and are used to designate limitations.

## Theory

Motion is divided to Falling and Collision. The former refers to the fall of  $n$  dominoes lying on each other before reaching  $n+1$ . The latter is related to the collision procedure of  $n$  and  $n+1$ .

### A) Falling

Forces applied to one of the dominoes are: 1) Normal force  $k-1$  and  $k$  2) Friction force  $k-1$  and  $k$  3) Normal force  $k$  and  $k+1$  4) Friction force  $k$  and  $k+1$  5) Weight 6) Normal force from surface 7) Friction force between domino and the surface.

$$I\ddot{\theta}(k) = \text{Torque}(\text{weight}, F(k-1), \mu F(k-1), F(k), \mu F(k)) \quad (1)$$

Dominoes always remain in contact. This results in an equation between height,  $h(k)$ , distance between right sides of  $k$  and  $k+1$ ,  $l(k)$ , width  $d(k)$ , and angle with surface  $\theta(k)$ .

$$\frac{h(k-1)}{\sin\theta(k)} = \frac{l(k-1) - \frac{d(k)}{\sin\theta(k)}}{\sin(\theta(k) - \theta(k-1))} \quad (2)$$

Using this equation, angular velocity and angular acceleration of  $k$  is found as a function of angle and angular velocity of the first domino. Equations (1) and second derivative of equation (2) form a system of  $2n$  linear equations and  $2n$  unknowns ( $F(k), \ddot{\theta}(k)$ ). This could be solved numerically to find angular accelerations of dominoes.

## B) Collision

Based on high speed videos, the following procedure is observed: As  $n$  hits  $n+1$ , some energy is wasted. Angular velocity of  $n$  and consequently preceding dominoes decrease.  $n+1$  obtains an angular velocity greater than the angular velocity of  $n$  after collision; therefore  $n$  and  $n+1$  separate. The large mass of the first  $n$  dominoes causes  $n$  to reach  $n+1$  soon again.  $n$  lies on  $n+1$  and the set of  $n+1$  dominoes continue to fall. Collision is immediate and the friction force between the domino and the surface is rather minor, hence conservation of momentum in horizontal direction is available. Restitution coefficient gives the relation between velocity before and after the collision. This coefficient is calibrated in experiments.

$$\sum_1^n m_i v_i = \sum_1^n m_i v'_i + m_{n+1} u \quad (3)$$

$$v'_{rel} = -\epsilon v_{rel} \quad (4)$$

## Simulation Program

Using MATLAB, a program is developed to fully simulate the motion. The angle and angular velocity of the first domino and properties of dominoes namely: height, width, distance, density and friction coefficient are inputs. In each iteration, angular accelerations of dominoes are calculated solving the system of linear equations discussed above. After, angular velocities and angles are updated. The program continuously checks if  $n$  has reached  $n+1$ . In case, that it has reached, it applies equations (3) and (4) to find velocities after collision. Then it uses first derivative of equation (2) to find angular velocities of preceding dominoes. After, it resumes the falling motion of first  $n$  dominoes and  $n+1$  separately until  $n$  reaches  $n+1$  again. The program assumes  $n+1$  to be in contact with  $n$  in the rest of the motion and upgrades the system to  $n+1$  dominoes. This process continues until the last domino reaches the surface.

## Experiments

Dominoes were made of Plexiglas in different heights and widths. Experiments were done on an abrasive to provide the non-slipping condition which was assumed in the theory. A screw was used to firmly tilt first domino to ensure free falling for the first domino with no initial angular velocity. Videos with 1000 frames per second were recorded. The dominoes, along with the background, were black and a thin white line was drawn on dominoes. Analyzing videos by MATLAB, white lines were detected and traced to measure the angle of dominoes. The time between initiation of the motion and first collision was used to calculate the initial angle. Giving this angle and zero value for initial velocity as inputs, the simulation program plotted the

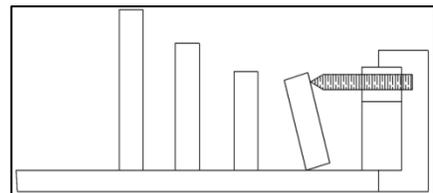


Figure 1: Setup Scheme

angle of each of dominoes vs. time graph. The same graph was plotted using video processed data. Precise agreement between simulated and experimental graphs demonstrated accuracy of the theory. Three chief experiments are presented.

**A) One Domino:** This experiment was done using one domino to check the falling procedure of the program.

**B) Two Dominoes:** Two identical dominoes were located. Falling time of the second domino was used to find the restitution coefficient. The great match verified equations of the collision.

**C) Increasing Height Dominoes:** Eight dominoes with different heights were placed in a row. The dominoes were each increased by 4 mm in height. The agreement between simulation program and video processing results in this experiment verifies theory's reliability.

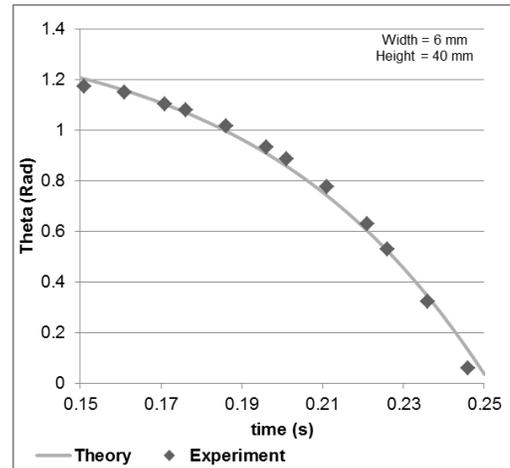


Figure 2: One Domino Experiment

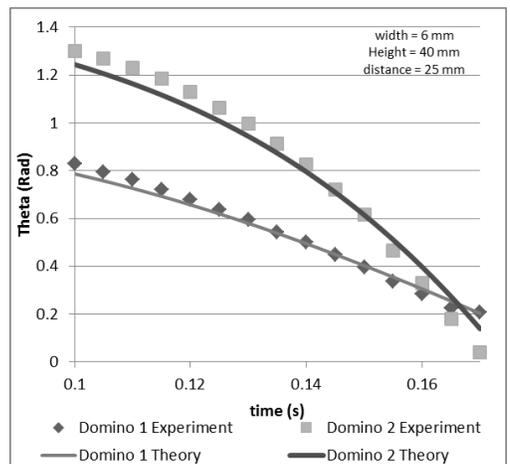


Figure 3: Two Dominoes Experiment

## Discussion

Considering the high agreement between experiments and the theory, the simulation program was used to demonstrate energies and limitations. An increasing height arrangement was studied. Dominoes, all with a constant width, were located in a fixed distance from each other. The height of dominoes were increased by a constant rate. In order to illustrate height increase effect, four situations were analyzed: identical height and increasing height rates of 2, 6 and 10 mm by each domino. The initial gravitational

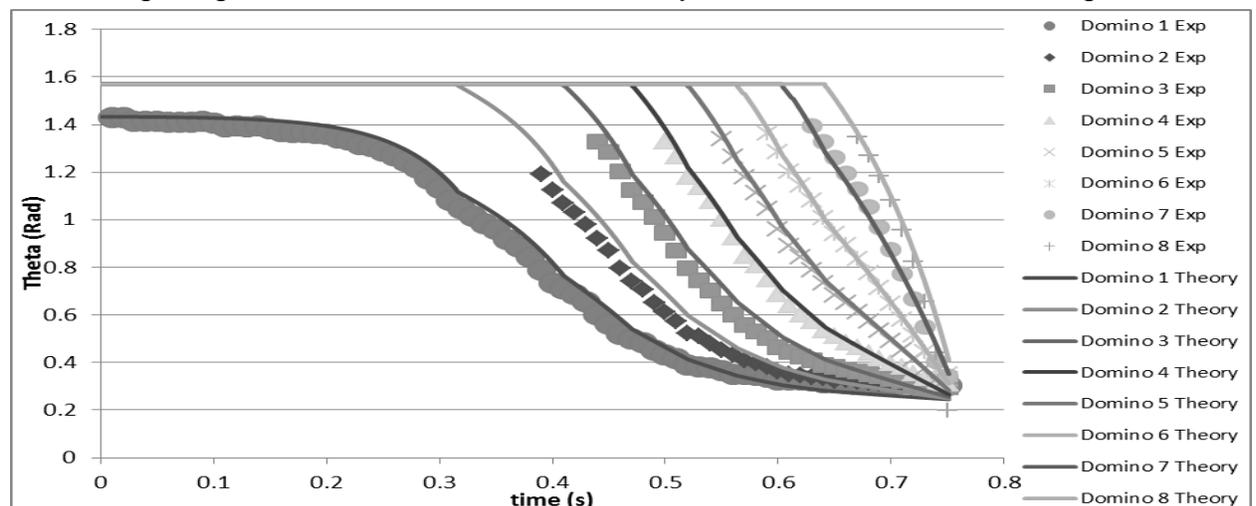


Figure 4: Increasing Height Dominoes Experiment

potential energy transforms into kinetic energy. Since dominoes are assumed to be stable on the surface, the kinetic energy is only rotational kinetic energy. In each collision, some energy is wasted. Using experiments, the restitution coefficient was calculated to be  $\varepsilon = 0.2$  for our set of materials.

Considering the collision of  $n$  and  $n+1$ , some part of the total kinetic energy remains in the first  $n$  dominoes, some transfers to  $n+1$  and the rest is wasted. Figures 6 and 7 illustrate this concept (e.g. In the collision of dominoes 20 and 21, in a 6 mm rate, 17 percent of the sum of kinetic energies before collision, will be transferred to 21<sup>st</sup> domino)

In case of identical dominoes, after approximately 5 collisions, the energy that the system gains due to the transformation of potential energy, becomes equal to the energy loss in the collision. Moreover, the time between collisions converges to a constant. Therefore, a wave of falling dominoes which moves with a constant velocity is observed.

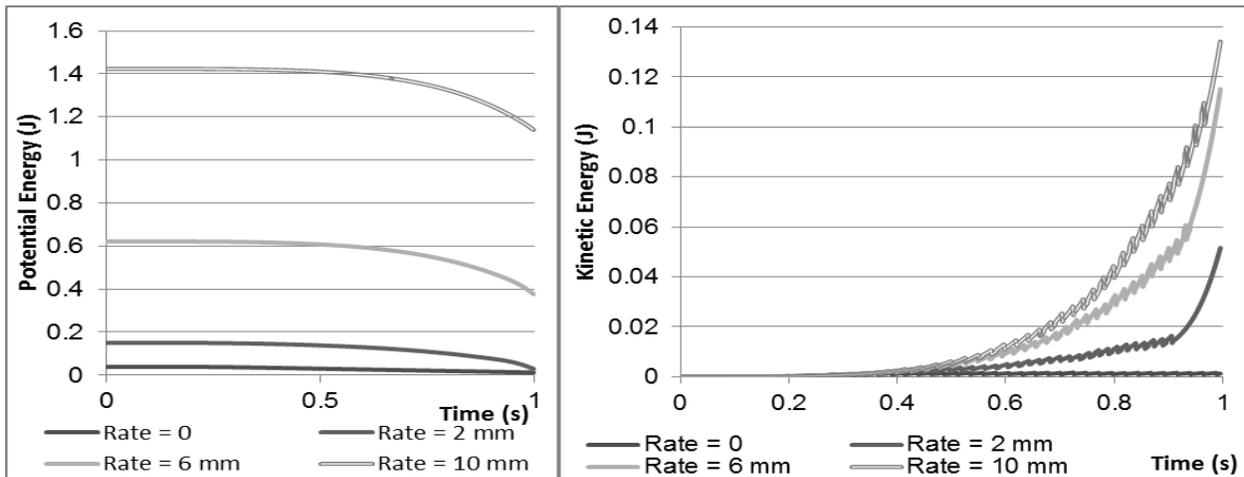


Figure 5: Potential Energy transforms into Kinetic Energy

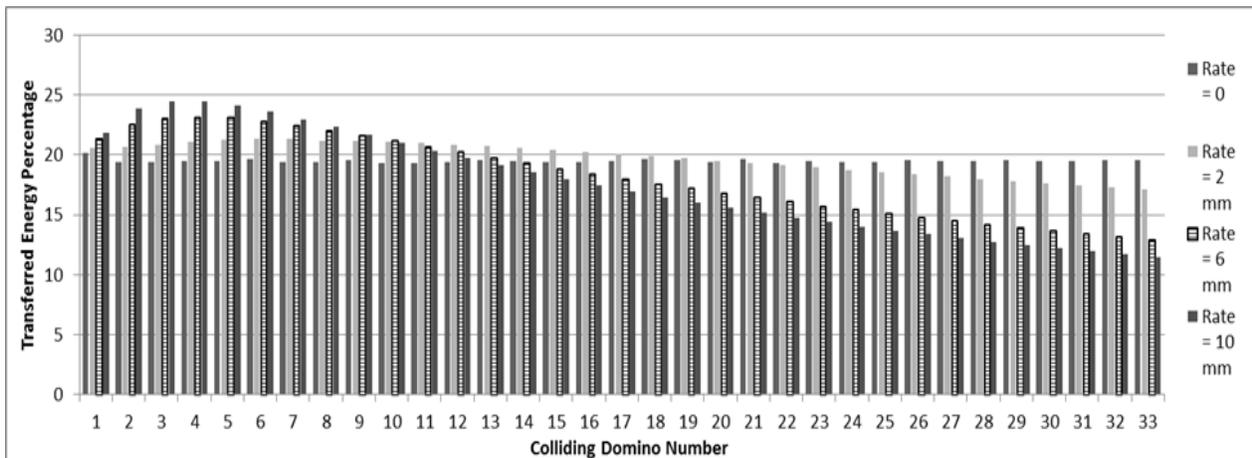


Figure 6: Transferred Energy Percentage at all collisions for 4 rates

## Limitation

The situation in which a domino withstands when it is hit could be defined as a limitation. Considering the potential energy of a domino (Figure 8-a), it requires an initial energy to be toppled. This energy is due to height increase of its center of mass which is supplied with the transferred energy. Hence, transferred energy should be greater than the initial potential energy. Such kinds of limitations are acquirable using the simulation program. If width of this domino exceeds a critical amount, the motion will cease. As the width increases, the motion stops sooner. For instance, figure 8-b shows the limiting domino against width for a particular set of dominoes. (e.g. if width is between 3.21 cm to 4.01 cm, the motion will stop in the 3<sup>rd</sup> domino)

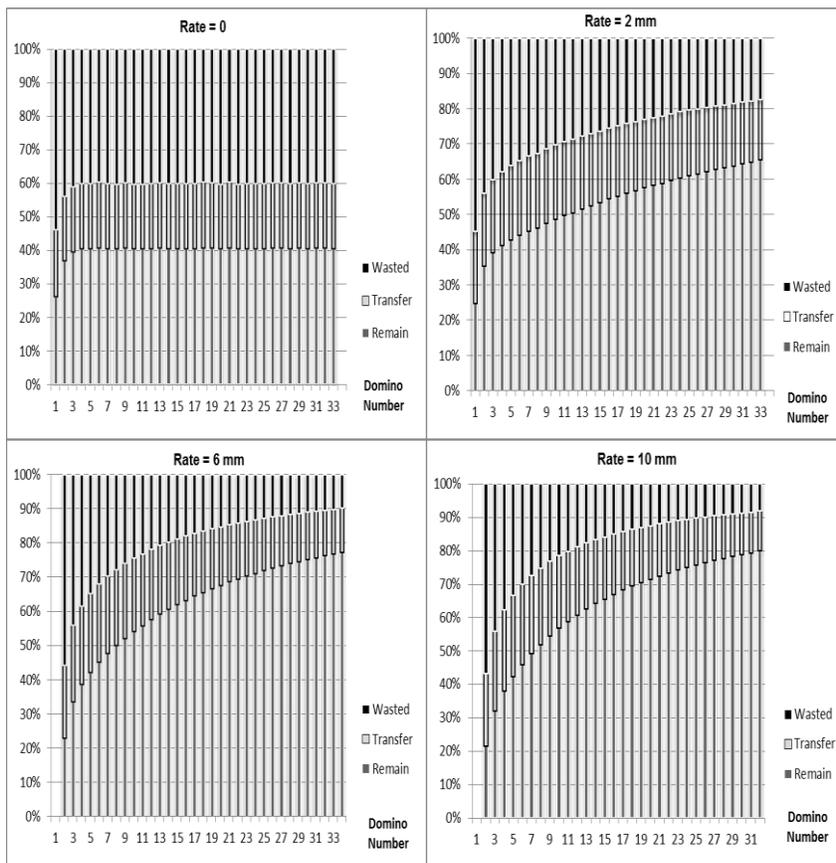


Figure 7: Remained, Transferred and wasted energy Percentages

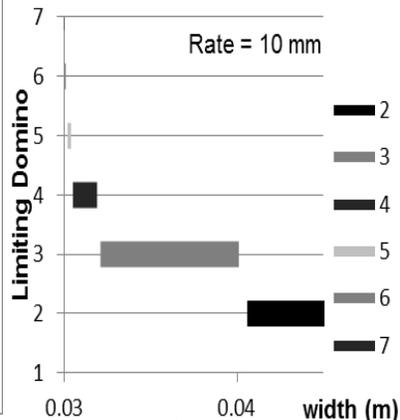
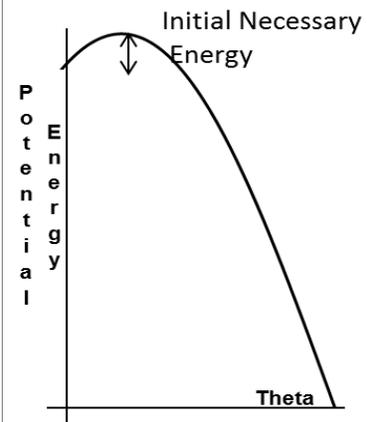


Figure 8: a) Potential Energy of a domino b) Limiting Domino against width

## Conclusion

The theory has been several times modified to present the best model. The acceptable matches between motion graphs ensure reliability of theory and the simulation program. The program works as well in every arrangement and height functions of dominoes. The simulation program, gives the motion of all the dominoes as a function of time. Therefore, all parameters including transfer rate, energies, collision times and etc. are found. Moreover, this program, is capable of recognizing any motion failure and consequently all the limitations.