**Supplementary material on:**

**DOMINO EFFECT MOTION INVESTIGATION: A NUMERICAL APPROACH**

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This document includes more information about the theory and modeling of the problem which is not appeared on the original paper.

**Physical modeling:**

Consider the forces applied to one domino (Figure 1):

These forces are:

*1)* Normal force *i-1* and *i* *2)* Friction force *i-1* and *i* *3)* Normal force *i* and *i+1* *4)* Friction force *i* and *i+1* *5)* Weight *6)* Normal force from surface *7)* Friction force between domino and the surface

The equation for the torque applied to the ith dominois:

More precisely:

Figure 1: Forces applied to one domino

 (1)

The other equation is the geometric constraint that is calculated by sinus law:

Second derivative of this equation against time results in:

 (2)

In better words, we can write two equations for each of dominoes.

 (3)

 (4)

Where C1(i), C2(i), C3(i), P1(i), P2(i), P3(i) are known constants.

The program has to solve this system of 2n equations (2 equations for each of dominoes) and 2n unknowns (, ) in each time iteration. It is obvious that F(n)=0, F(0)=0.

Therefore if we make these matrices (examples of 4 dominoes are shown):

M =

 0 0 C2(1) 0 0 0 0 0

 0 P1(1) 0 P2(1) 0 0 0 0

 0 0 C1(2) 0 C2(2) 0 0 0

 0 0 0 P1(2) 0 P2(2) 0 0

 0 0 0 0 C1(3) 0 C2(3) 0

 0 0 0 0 0 P1(3) 0 P2(3)

 0 0 0 0 0 0 C1(4) 0

 0 0 0 0 0 0 0 P1(4)

This equation is available:

M=X\*B

So X=B\*inverse (M) and F(i) and is found for all i<=n (i.e solving a 2n\*2n matrix)

After solving this matrix in each of iterations, the program updates amounts of for all i<=n by using the following equations:

* (5)
* (6)

**Collision:**

Several methods for modeling the collision were assumed and tested. The best method is presented. The equations used are Reservation of momentum in x direction and using the restitution coefficient. Reservation of momentum in x direction is available since the collision time is pretty small. We have:

 (7)

Which could be rephrased:

 (8)

First derivative of the geometric constraint against time gives relation between angular velocities of any 2 neighboring dominoes:

 (9)

This equation indicates that if decreases by coefficient α after the collision, will also decrease by the same amount. Therefore, if it could be inferred that for 0<i<=n

Re-writing equation (7) we get:

 (10)

For simplifying we call S=. Equation (8) gives

S(1-α)= (11)

The other equation is restitution coefficient. The relative velocities of n and (n+1) get –e times (e<1) after the collision.

 (12)

The coefficient e, is calibrated in experiments. Solving equations (11) and (12), with two unknowns of **α** and two important parameters are calculated. The program updates from 0 to its new amount and also multiplies by the coefficient α for 0<i<=n.