## PROBLEM № 12 LEVITATING SPINNER

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Problem formulation: "A toy consists of a magnetic spinning top and a plate containing magnets (e.g. "Levitron"). The top may levitate above the magnetic plate. Under what conditions can one observe the phenomenon?"

For experiments was used "Levitron". This toy consisted of plate, spinner and ring weights.

Our goal is to investigate this system of magnets. Make model which would explain this phenomenon qualitatively and quantitatively.

Possible motions (degrees of freedom) of spinner should be observed first. Experimental set up has axial symmetry around vertical axis, so we can observe only one vertical plane. There are 3 possible movements of the spinner:

- Vertical;
- Horizontal;
- Angular (turning of spinners axis).

It comes out that spinner should be stable in this "dimensions". This mean that there should exist forces (in some points of space) acting on the spinner which are:

- Equal to spinner's weight;
- Move spinner to the axis of experimental set up's symmetry;
- Prevent turning of spinners axis;

Nature of some forces is clear. Forces of magnet interaction should exist. They are caused by magnets in plate and magnet in spinner. As approximation of magnet we can use circular current. For such current we can calculate magnetic field in each point of space using Bio-Savart law:

$$\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I \cdot d\vec{l} \times \vec{r}}{\left|\vec{r}\right|^3}$$

And here is this law in our model:





We'll come back to model later. Now let conceder different system of magnets.

1. Two repulsing magnets. Top magnet is free, down is fixed. Let suggest that top magnet has small deflection from axis of symmetry (which is vertical) of down magnet, and small angle between its axis of symmetry and vertical. Lets conceder forces acting on top magnet:



As it's shown on the picture:

- Here is vertical forces which can be equal to weight;
- Also vertical and horizontal forces creates moment of forces which turns the spinner;
- And horizontal forces moves the spinner away from axis of down magnet;
- 2. Two attracting magnets. All other conditions are the same as in point



From the picture:

- Horizontal forces move the spinner to the axis of down magnet;
- But vertical forces are still turning the spinner;

So we should combine attractive and repulsive magnets. This combination provides us vertical and horizontal stability.

To check how this combination was made in "Levitron" dependence of force against some space dimension is necessary.

Spinner was fixed on vertical axis (same to axis of platform's symmetry) to prevent its turning round, and vertical force against vertical coordinate was measured:



Plot (Vertical Force against vertical distance). Force is sum of force of magnet interaction and weight. So F=0 is equilibrium. And second equilibrium ( $Z\approx3.5$ ) is stable.

After fitting using mathematical model we could calculate approximate currents:



Next using model it's possible to fide zone of horizontal stability:



Plot (Horizontal Force against horizontal deflection, vertical coordinate is constant). Where Force is lower than 0 spinner attracts back to 0 deflection.

Where Force is lower than 0 spinner attracts back to 0 deflection. So it's zone of horizontal stability. Taking together vertical and horizontal stability it's possible to plot zone of stability in space:



As you can see zone of stability in vertical is from 2 to 3.5 approx. In reality in such a big zone levitating can't be started correctly because when spinner got to point Z=2 it starts accelerating up until it reach point z=3.5 but by this moment it has big enough velocity to get higher than the point of the end of the horizontal stability zone. So it falls. We mentioned in the beginning weights for the spinner. They are necessary to avoid situation which was described. Now let's correct previous plot including additional mass in calculation of vertical zone:



So the part of the problem connected with vertical and horizontal stability is solved. Let's come to the part connected with turning round the spinner.

Having mathematical model we can easily find out moment of force which cause rotation of the spinner:



Plot (Moment of Forces against angel of deflection ( $\alpha$ =0 means axis of spinner is vertical).

As you see from the plot equilibrium is unstable so with small deflection spinner will turn round.

Let's conceder usual spinner :



ωll and ωL -projection of angular velocity along spinner's axis and perpendicular; III and IL – moment of inertia for rotation and turning; s - unit vector along spinner's axis; M - moment of acting force; L - angular momentum;

If we know moments of inertia and angular velocities we can calculate angular momentum:

momentum:  $\vec{L} = I_{\parallel}\vec{\omega}_{\parallel} + I_{\perp}\vec{\omega}_{\perp}$ Then from unit vector s and its first derivative it's possible to express  $\omega L$ :  $\vec{\omega}_{\perp} = \vec{s} \times \dot{\vec{s}}$ Then knowing that :  $\vec{L}=\vec{M}$ We can express M:  $\vec{M} = I_{\parallel}[\dot{\vec{\omega}}_{\parallel} \times \vec{s}] + I_{\parallel}[\vec{\omega}_{\parallel} \times \dot{\vec{s}}] + I_{\parallel}[\vec{s} \times \ddot{\vec{s}}]$ 



 $\ddot{\vec{s}} = -\Omega^2 \cdot \vec{r}$ Let's transform this equation knowing that:  $\dot{\vec{s}} = \vec{\Omega} \times \vec{s}$  $\Omega$  – angular velocity of precession;

We'll get square equation:  $I_{\perp}\Omega^2 \cos \alpha - I_{\parallel}\omega_{\parallel}\Omega + M = 0$ 

It will have solution only if:  $I_{\parallel}^2 \omega_{\parallel}^2 > 4MI_{\perp} \cos \alpha$ 

In other case spinner would fall. So we have to put in moments of forces we calculated in mathematical model and express critical angular velocity:

$$\omega_{theor} \approx 22 \frac{rad}{s}$$

Then using HS camera we recorded levitation and calculated critical angular velocity from experiments: rad

$$\omega_{exp} \approx 35 \frac{7aa}{s}$$

As you can see experimental  $\omega$  is bigger than theoretical one. It happen so because in model we use approximation that spinner levitates exactly on the axis of set up while in reality it has horizontal oscillations due to which spinner gets to another part of magnetic field and forces acting on it is differ from forces which acts on the spinner when it's on the axis of set up.

Conclusions:

- Magnetic field of plate should :
  - 1. Cause force to compensate weight of the spinner,
  - 2. Move levitating magnet to the center;
- Field of plate can be modeled as field of 2 current circles;
- This model gives us ability to "calculate" this system by one simple measurement;
- Magnetic field causes forces which turn round the spinner;
- Rotation of the spinner prevents turning round;
- Increasing of angular momentum increase the spinner's stability;
- We can get critical angular velocity from accurate theory of gyroscopes (22 rad/s) and from our experiment movies (35 rad/s);
- Spinner should have:
  - 1. Magnet;
  - 2. Optimal moment of inertia;
  - 3. Optimal ratio between mass and "current" in the magnet;

## References

- [1] "General course of physics" vol. 5 "Mechanics" D. Sivuhin;
- [2] "Simple theory for the Levitron" Jones, Washizu, Gans;
- [3] "Diamagnetic levitation: Flying frogs and floating magnets" Simon, Geim;