

IYPT 2011 PROBLEM 11: FINGERPRINTS

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Introduction

A glass is filled with liquid and held with hand. If looked from above at the inner walls of the glass, sometimes the only thing visible through the walls is a very bright and clear image of patterns on fingertips. In this paper, this interesting phenomenon was studied and explained.

There is a critical angle that separates two different images. For smaller angles, we see blurred image of a whole finger and its surrounding. For greater angles, only clear image of a fingerprint can be observed. The existence of a critical angle related this phenomenon to total reflection, which became our main idea in explaining it. Another two properties of light were important: refraction of light and optical tunneling. The phenomenon of optical tunneling distinguishes the ridges and the grooves of a fingerprint, since they do not affect the light in the same way. The relevance of Fresnel's equations was also studied. It was investigated how the image and its intensity changes over some angle interval around the critical angle.

The secret of such bright and clear image of a fingerprint lies in finger's relief structure. Fingers do not have smooth, even surface – they consist of ridges and grooves (fig.1). Grooves have the order of magnitude 10^{-4} m, which is much greater than the wavelength of light (10^{-7} m). Therefore, when pressing the glass with fingers, we only touch the glass with our ridges, while the gaps between grooves and glass are filled with air. The simplified model was made using small pieces of paper glued with adhesive tape (fig.2). Adhesive tape had the role of ridges (due to big adhesion with the surface) and the paper had the role of grooves (smaller adhesion → less attached to the glass). When seeing fingerprints, on this model only adhesive tape can be observed (fig.3a). If we see the whole finger, not just the fingerprint pattern, on this model we see both adhesive tape and the paper (fig.3b). The model was used for easier measurements and parametrization of the problem.

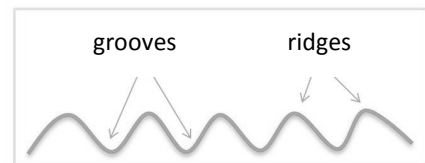


Figure 1. Enlarged fingerprint section

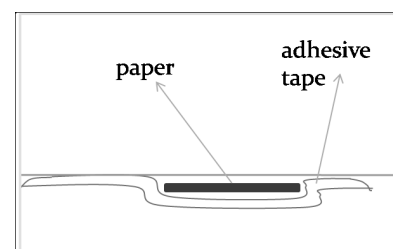


Figure 2. Fingerprint model



Figure 3a. Fingerprint visible

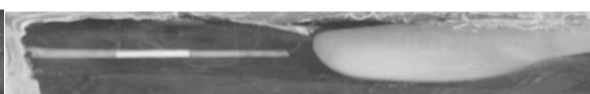


Figure 3b. Fingerprint not visible

Two main questions were studied in this paper: Why nothing outside the glass is visible? And why such clear image of a fingerprint is visible? The main goal of this paper was to answer these questions and to give a good understanding of the phenomenon.

Theory

If a light ray comes outside of the glass, it can be totally reflected from the surface of the liquid, which means it will not get to the observer's eye (fig.4). We used Snell's law ($n_i \sin \alpha_i = n_t \sin \alpha_t$, where i stands for "incident", and t for "transmitted") for air-glass and glass-liquid barrier and the law of total reflection for

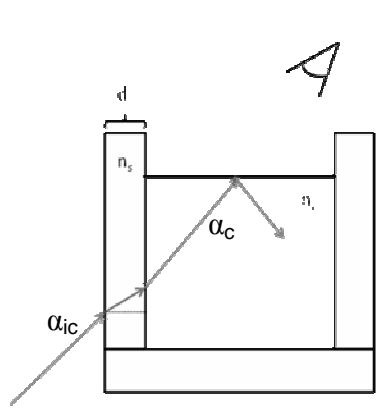


Figure 4. Light ray from outside not coming to the observer's eye

liquid-air barrier ($\sin \alpha_c = \frac{1}{n_L}$, where α_c is the critical angle at which total reflection occurs, and n_L is the index of refraction of liquid). With little trigonometry we obtained

the expression $\sin \alpha_{ic} = \sqrt{n_L^2 - 1}$. Angle α_{ic} is the critical incident angle at which total reflection occurs. For every incident angle smaller than this one, the light will get totally reflected and the observer will not see the light ray. In this case, since the whole ray is reflected, there is not any

energy loss. For angles greater than α_{ic} , light ray will be refracted at the liquid-air surface and will get to the observer's eye. When the light is refracted and goes to another medium, there is always some percentage of light that gets reflected. Due to conservation of energy, intensity of a light ray that leaves the water will be smaller than the intensity of the incident ray.

When we observe the fingerprint pattern, the distance d between the ridges and the glass is very small, almost negligible ($d \approx \lambda$, wavelength of light). This causes the phenomenon of optical tunneling. Light travels in optically denser medium and reaches less dense medium at the angle greater than the angle of total reflection. In the normal conditions of only two media, the light is totally reflected back to the first medium. But, if the third medium is put very close to the first two media, ($d \approx 10^{-7} \text{ m}$), the light will not get totally reflected, but will manage to pass to the second and the third medium. This is because, at the end of every medium, an exponentially decayed light wave exists. If there is the third medium put very close, electric and magnetic oscillations will pass to the third medium before they exponentially disappear in the second medium. Light that incidents glass-air barrier (Fig.5a) will not get totally reflected if there is a ridge very close to the glass. In this case light passes through and gets absorbed and scattered by the ridge. It comes to the observer's eye with smaller intensity, same as the light rays from surrounding behind the glass, explained above. The distance between grooves and the glass is much bigger, thus optical tunneling does not appear because exponential wave in the air decays much before it would reach the groove. Light totally

reflects and comes to the observer with the full intensity. Thus, fingerprints are visible if the light incidents the glass-air surface with the angle greater than the critical angle for

total reflection (for glass $n_G=1.5$, $\sin \alpha_{Gc} = \frac{1}{n_G} \rightarrow \alpha=41,8^\circ$). By using this law and general Snell's law ($n_L \sin \beta = n_G \sin \alpha$), we can theoretically determine critical observing angle α_0

: $\cos \alpha_0 = \sqrt{n_L^2 - 1}$. For every angle greater than α_0 , fingerprint pattern will be observed.

We see that the only relevant parameter for the occurrence of this phenomenon is index of refraction of a liquid (fig.5a). With more geometrical approach, we can determine the

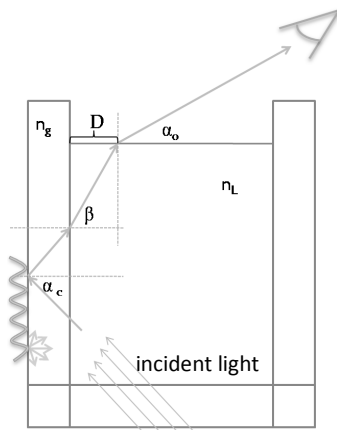


Figure 5a. Deriving expression for the critical angle

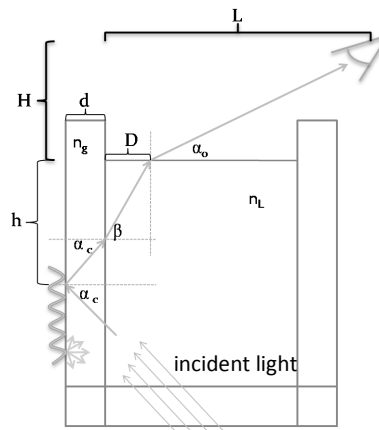


Figure 5b. Experimental determination of the critical angle

expression for D , the distance at which the light ray leaves the water:

$D = (h - dtg\alpha_{cc})tg\beta$ which is then used for the experimental determination of the

observing angle: $tg\alpha_0 = \frac{H}{L - D}$ (fig.5b), where H is the height of observer's eye and L is the observer's distance from the pattern. By doing measurements it was possible to compare the results with the theoretical expression.

The problem with this phenomenon is that it is NOT binary. That means that the critical angle can only be approximately determined, because the intensity of the image changes from maximum to minimum value over some interval of angles. For this purpose, Fresnel's equations were proposed in the paper. They give information about total intensity of reflected and transmitted light for a given incident angle and indices of refraction. By using these equations and connecting the angle of incidence (from air to glass) with the observing angle, we made theoretical graph showing the transmittance in dependence on the observing angle, i.e. how the visibility of finger changes.

Experiment

The experiment consisted of a few parts: measuring indices of refraction for 5 liquids (water with different concentrations of sugar), determining critical observing

angle, and determining the intensity of light for different observing angles.

Indices of refraction were determined using a semi-circular vessel in which the liquid was put. With three needles one light ray was defined, and by measuring the incident and transmitted angle, one could determine the index of refraction.

For determining critical observing angle, experimental setup was made.

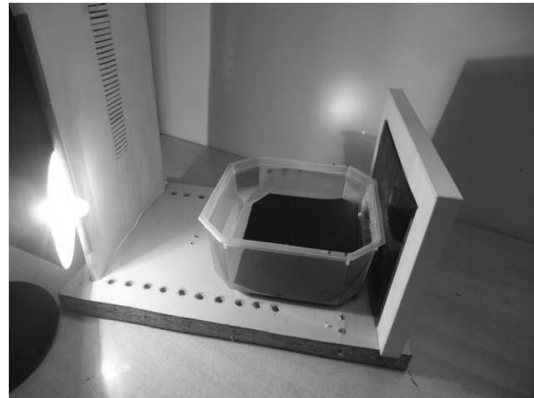


Figure 6. Experimental setup

Rectangular vessel with water had one glass side for observing the patterns. This setup was observed through “windows” which enabled us to control the height of observing. For every height, the distance from the pattern was determined (for critical observing angle). From the height-distance graph, observing angle was determined. This was done for every measured index and compared to the theoretical result.

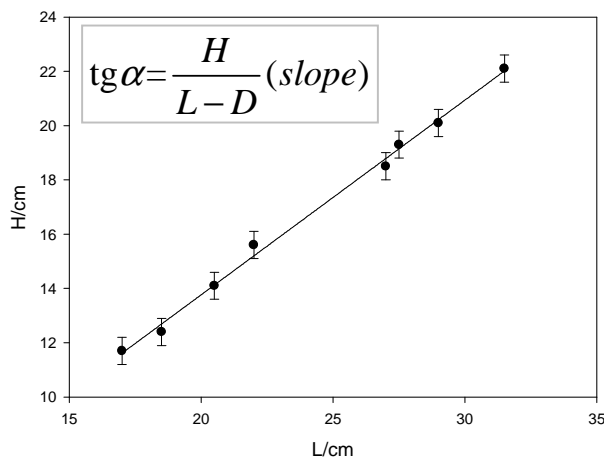
Relevance of Fresnel’s equations was investigated by taking a photo of an image and analyzing it in a computer program. The graph showing the intensity of light in dependence on the depth of observed image was obtained. Dependence of depth on the observing angle can easily be derived using a little geometry and trigonometry → by transforming depth axis to observing angle axis, we got the graph that was compared to theoretical graph that we got using Fresnel’s equations.

Results

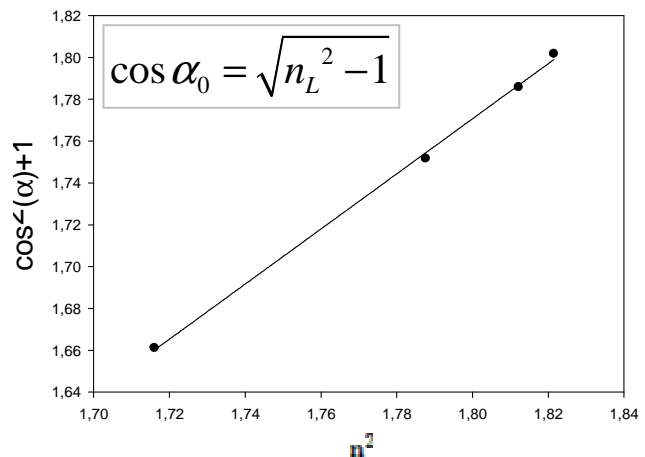
5 indices of refraction were determined. For every index, the critical angle was determined experimentally and then compared to the theoretical value.

Indices of refraction
$n_1 = 1,31 \pm 0,02$ (pure water)
$n_2 = 1,34 \pm 0,01$
$n_3 = 1,346 \pm 0,014$
$n_4 = 1,350 \pm 0,03$

Table 1. Indices of refraction



Graph 1. Height-distance dependence

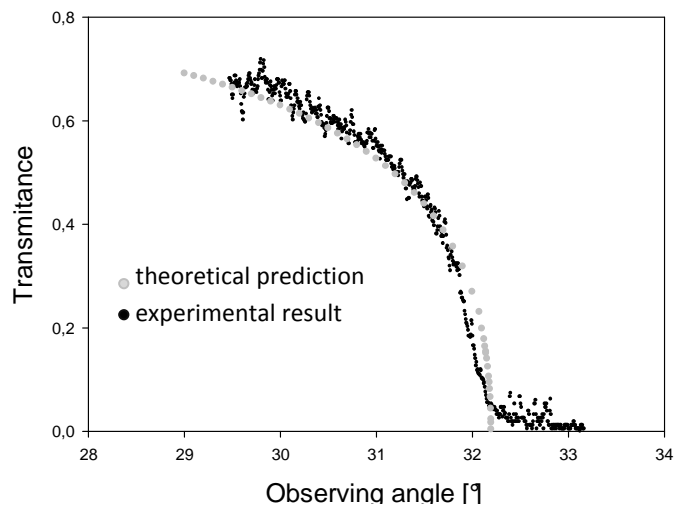


Graph 2. Critical angle-index of refraction dependence

From the first graph we obtained critical angle as a slope of the graph. On the second graph we can see the relation between critical angle (little transformed expression) and the index of refraction. From theory, we expected the slope to be 1, but we got the experimental slope 1,3.

Intensity measurement was done for a bigger piece of paper – that way we could observe the effect for different angles at the same time and compare how transmittance changed with depth → observing angle.

We got the intensity in dependance on depth of the pattern. By transforming the depth into observing angle, the graph was obtained that could be compared to theoretical graph. We got the dependance of intensity (transmittance) on the observing angle. We can see that Fresnel's equations plays role for this phenomenon and that intensity change happens in interval of 3°. These 3° goes into our estimated error in determining critical angle. That is why, although transmittivity change exists, we can not actually percieve it with our own eye successfully and for us the phenomenon seems binary, although it is not.



Graph 3. Intensity of light coming from the outside of the glass – grey: theoretical curve, black: experimental curve

Conclusion

In this paper the phenomenon of clear image of fingerprints on a glass of water was studied and explained. The main reasons for this effect to occur are total reflection and optical tunneling. When the angle is greater than the critical angle of total reflection, on the ridges light will tunnel and be absorbed and then scattered by the finger surface. This we see as light with lower intensity and in skin colour. On the grooves, light will only get totally reflected and will come to the observer with the whole intensity. That we do not see as skin colour, but as the reflected light from the bottom of the glass (from the light's direction). Nothing outside of the glass is visible because of total internal reflection on the water surface. The light that manage to pass through and get to the observer's eye has a very low intensity due to lots of refractions and energy losses. The phenomenon was described using formulas which were then validate by experimental measurements. The error exists, but it is not big and experimental results can be compared to theoretical predictions. The change in image intensity was also studied

and it was shown that transmittance of the light from outside of the glass (from the finger) changes from maximum to minimum value over a few degrees near the critical angle.

References

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