23rd IYPT Problem 16: Rotating Spring

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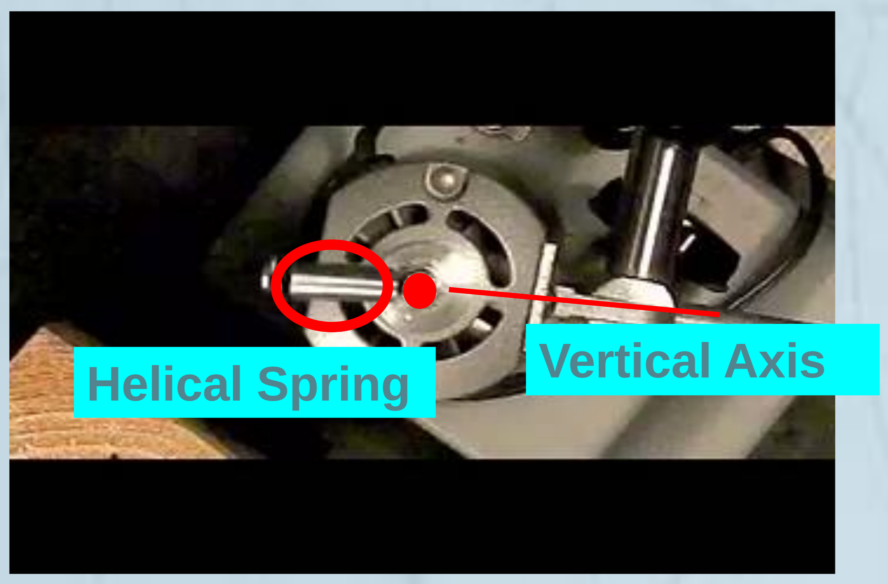
Abstract

A helical spring is rotated about one of its ends around a vertical axis. Investigate the expansion of the spring with and without an additional mass attached to its free end.

Attached to an axis, the spring is allowed to rotate freely; this can be easily explained by Newton's second law of motion and the equilibrium of forces. We place our focuses mainly on the distance between each ring of the spring, and we can observe that, as the additional mass increases the distribution of the ring becomes more and more uniformed. By means of calculus and related computer calculations, we can then discover how different parameters influence the results differently.

Experimental Setup

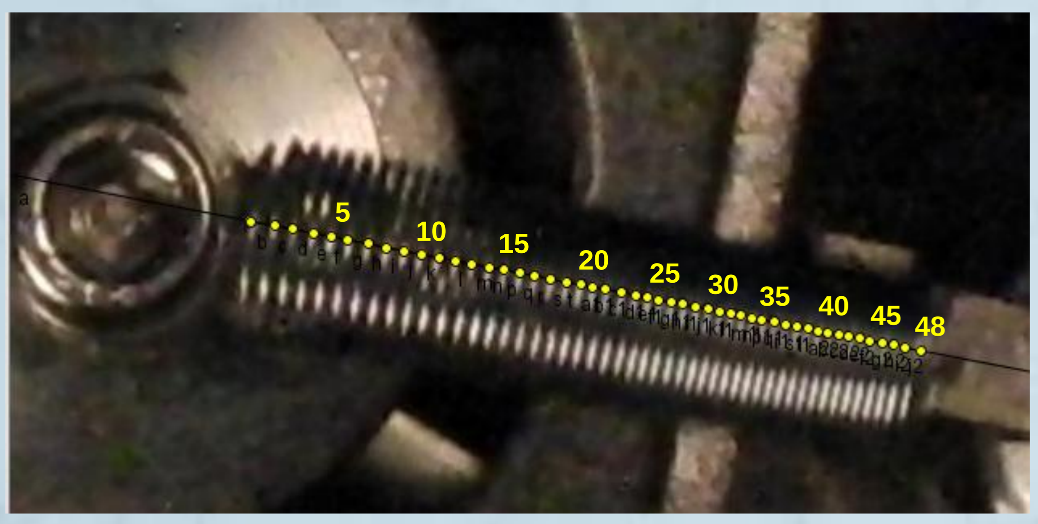


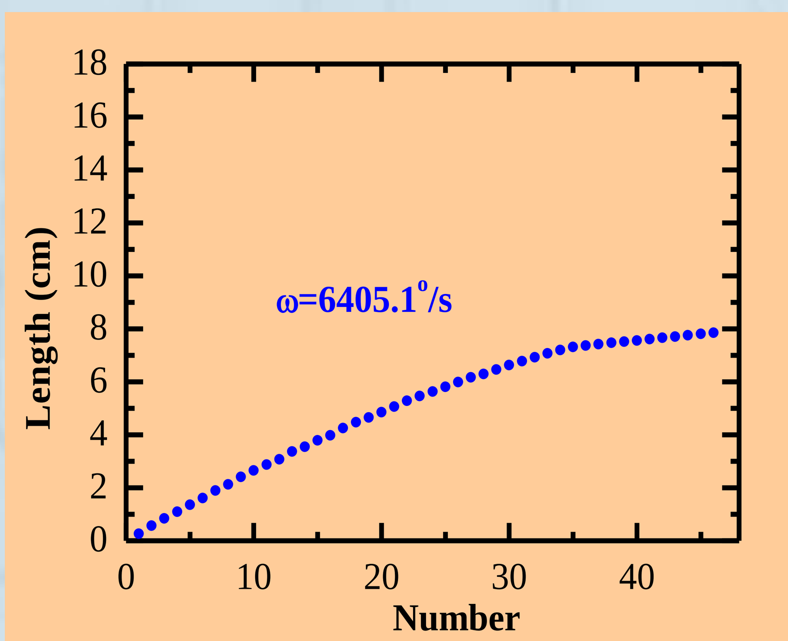
Experimental Details

- m (mass of spring): 2.3 g
- M (additional mass): 1, 2, 5 g
- ω (angular velocity)
- l_o (original length): 2.6 cm
- k (force constant): 2.5 N/m

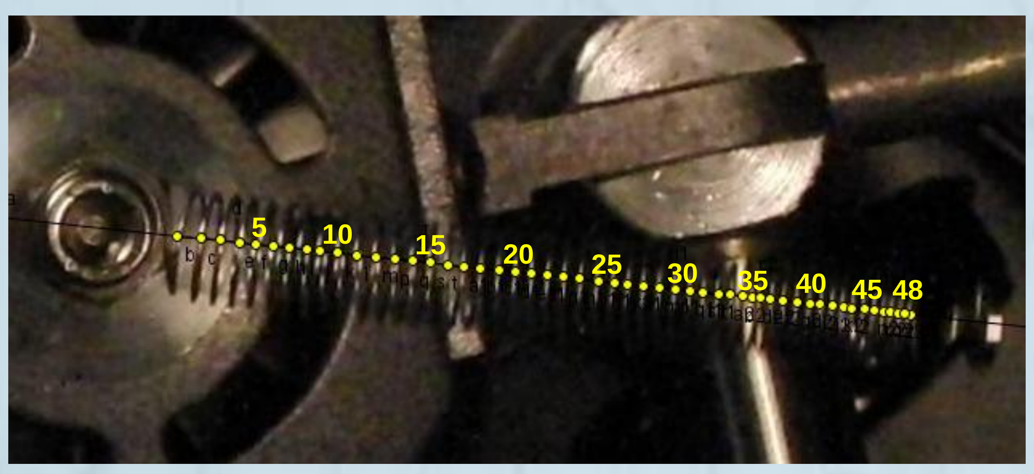
Experiment

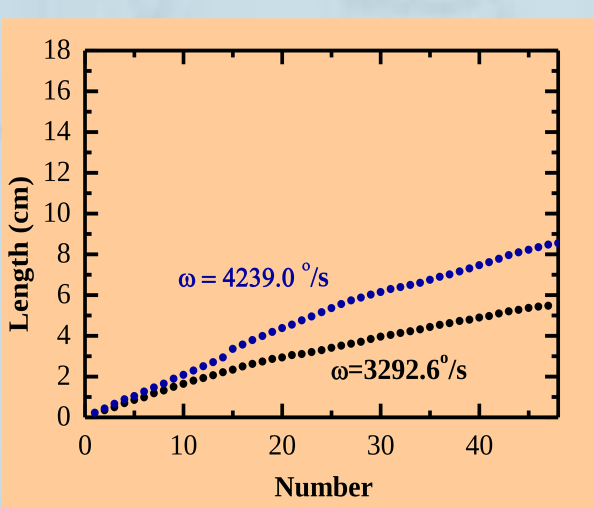
■Spring without mass

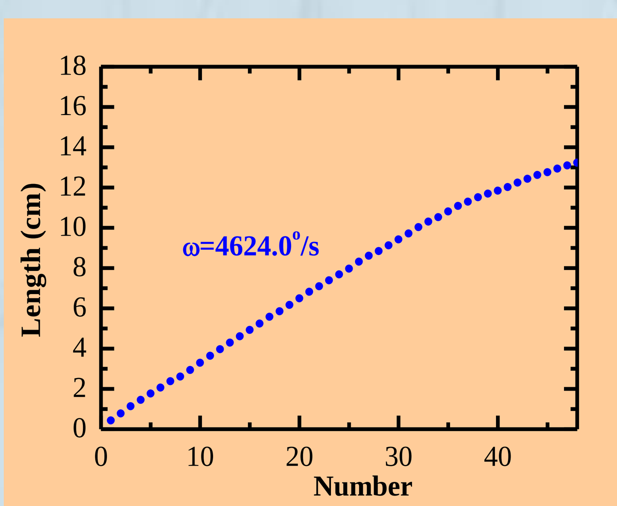


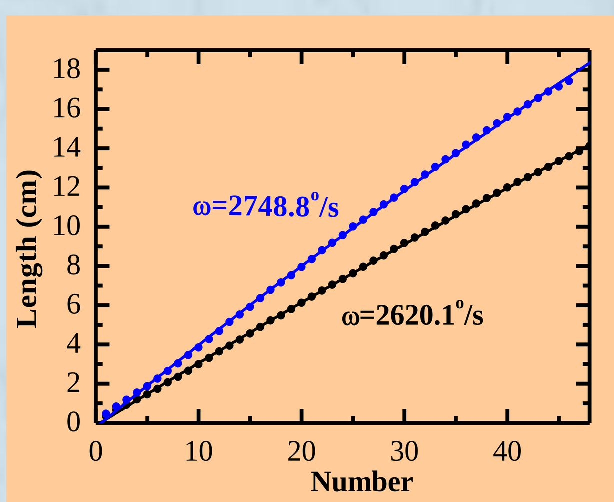


■Spring with mass

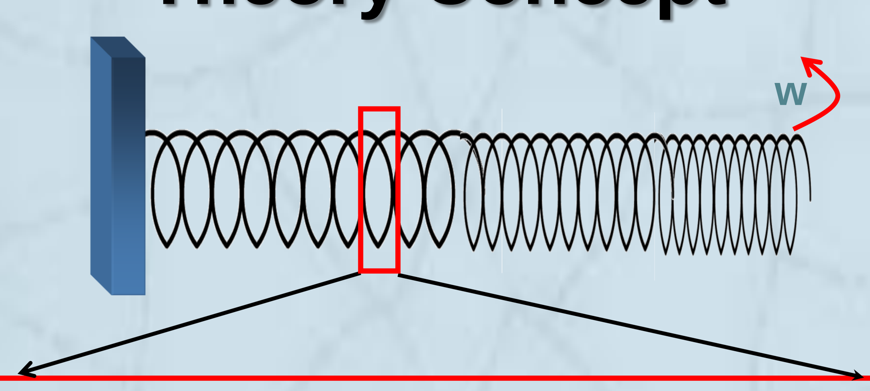




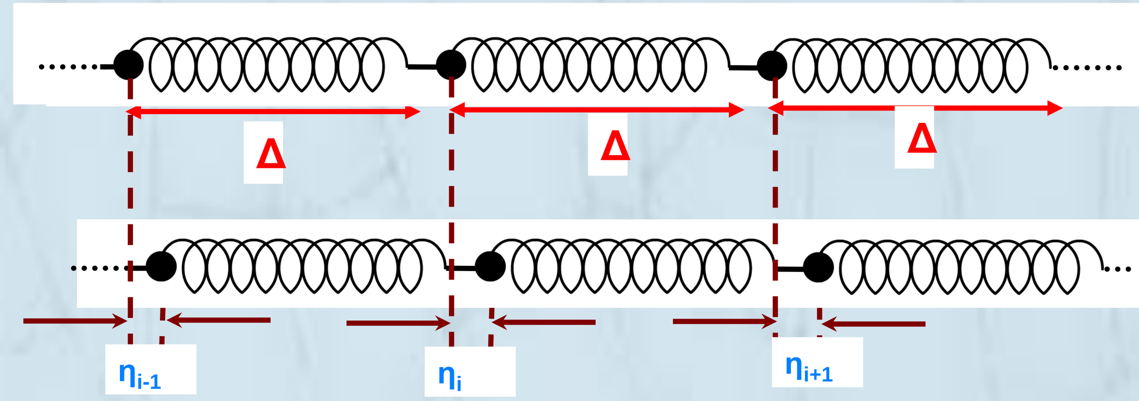




Theory Concept



Net force of Spring = Centrifugal Force



Centrifugal Force:

$$F_c = m\cdot\left(i\cdot\Delta+\sum_i \eta_i\right)\omega^2$$

Force to the right:

$$F_r = k(\eta_{i+1}-\eta_i)$$

Force to the left:

$$F_l = -k(\eta_i-\eta_{i-1})$$

Newton's second law of motion in equilibrium:

$$\Rightarrow F_r + F_l = F_c$$

Continuum Limit: $\Delta \to 0 \quad i\Delta \to x$

$$k\Delta \to Y \qquad \frac{\mathrm{m}}{\Delta}\to \mu$$

$$\frac{\eta_{i+1}-\eta_i}{\Delta}\to \eta'(x) \qquad \frac{\eta_{i+1}-2\eta_i+\eta_{i-1}}{\Delta^2}\to \eta''(x)$$

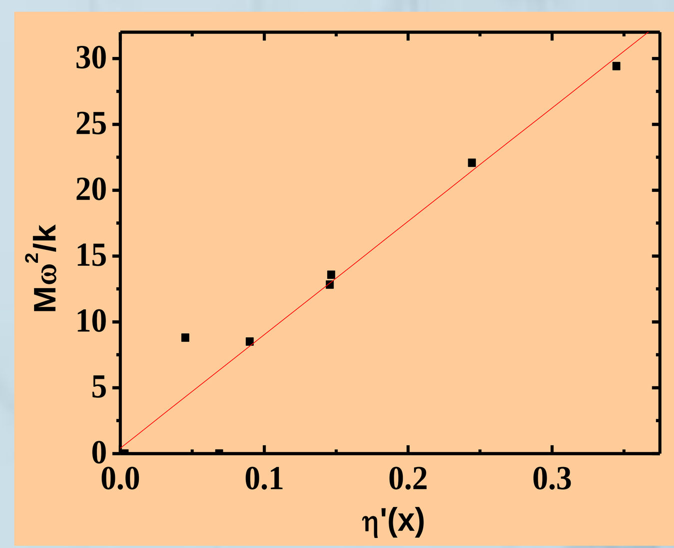
General Solution:

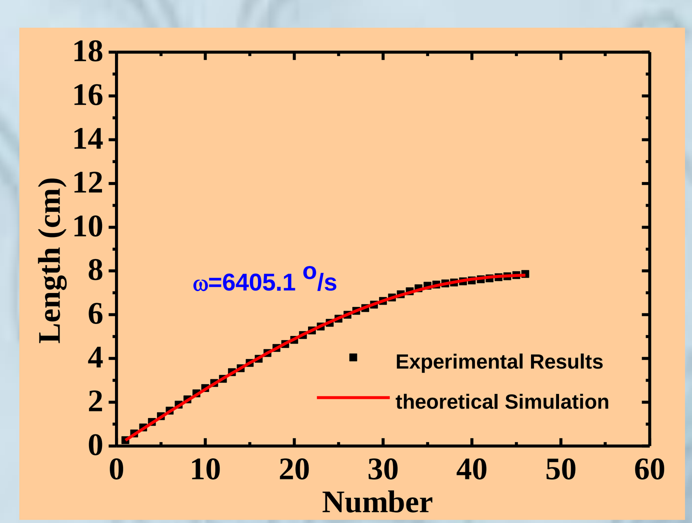
$$\eta(x) = -x + A\sin\left[\frac{x}{x_o}\sqrt{\frac{m}{k}}\omega+\theta_{\max}\right]$$

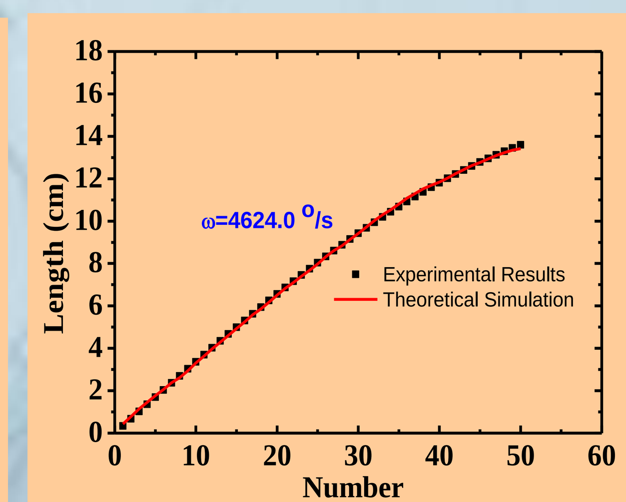
Sinusoid formula

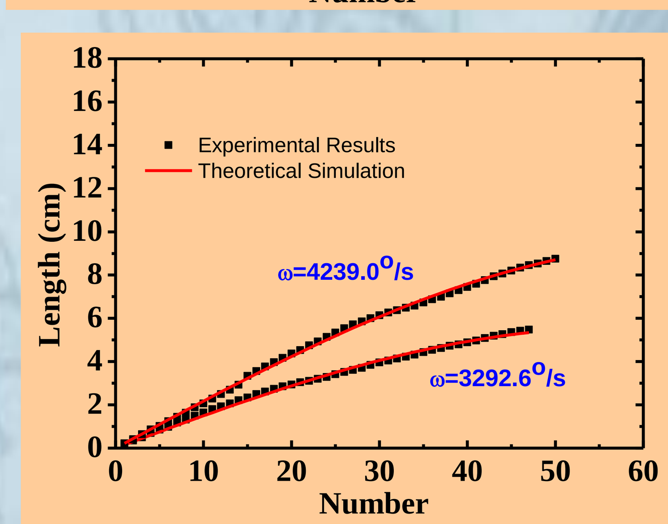
$$\eta'(x) = \frac{M\cdot\omega^2}{kx}\frac{[x_o+\eta(x_o)]}{x_o}$$

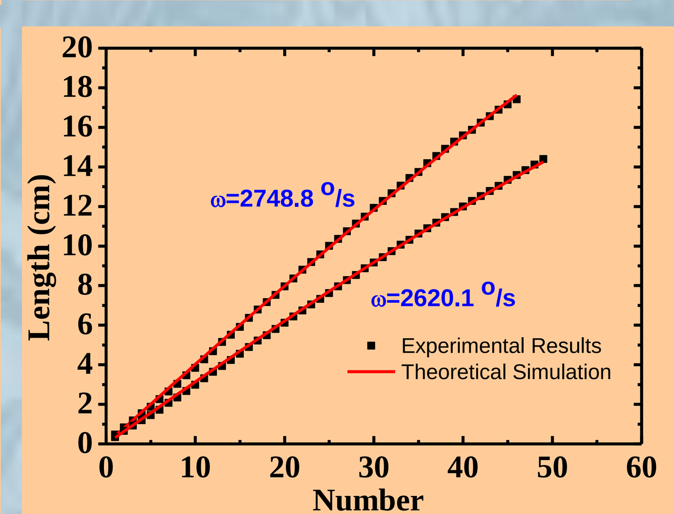
$$\eta'(x_{end}) \propto M\cdot\omega^2$$











Theory Concept

- **Without mass**
  Distribution of the spring is nonuniform
  Denser at further end of the spring
- **With mass**
  More uniform distribution of the spring
- **The variation of the length of spring at the end increases as:** $\eta'(x_{end}) \propto M\cdot\omega^2$