23rd IYPT Problem : Magnetic Spring

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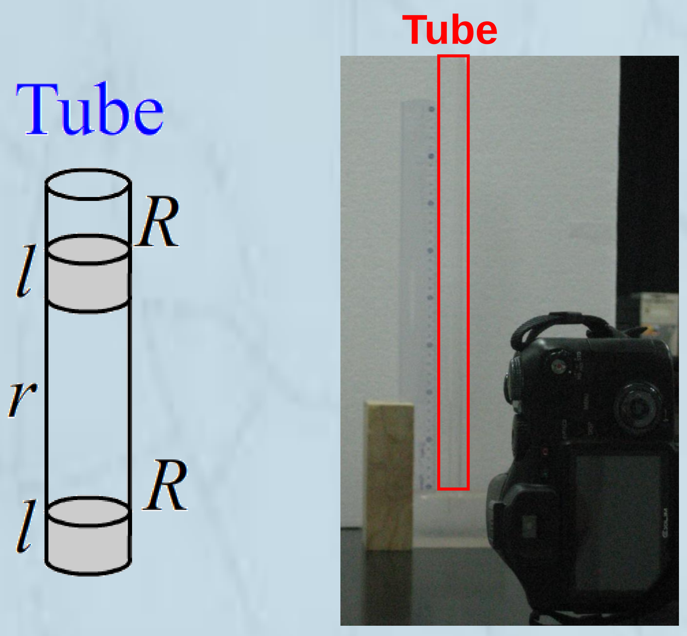
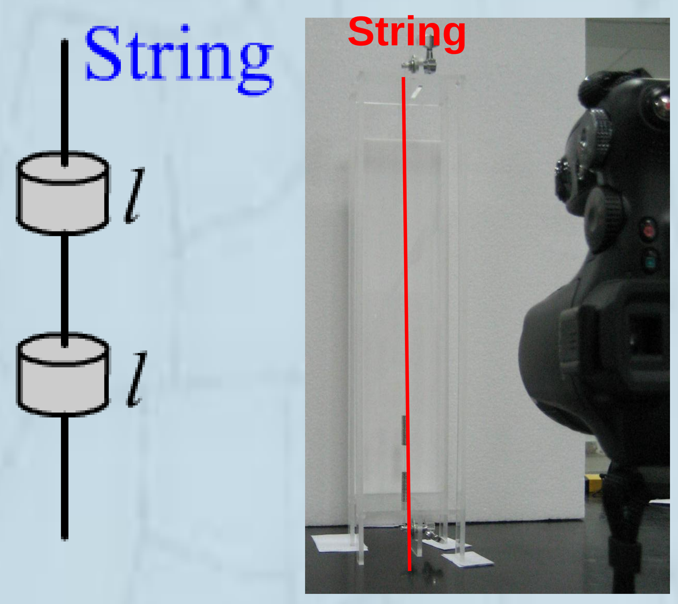
Abstract

Two magnets are arranged on top of each other such that one of them is fixed and the other one can move vertically. The oscillations of the magnet were investigated.

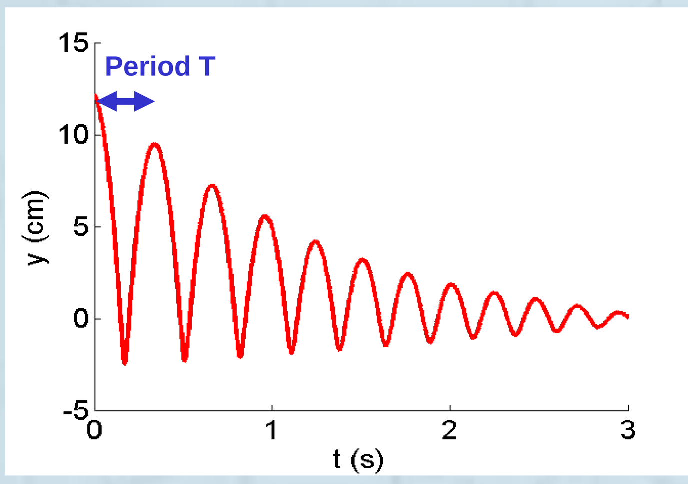
When the system is in equilibrium, the magnetic force is balanced with gravitational force. If the magnet is perturbed from its equilibrium position, it exhibits different oscillation patterns depending on the initial conditions and the resistive forces. The magnets can be modeled as magnetic dipoles, and the interaction between them is the dipole-dipole interaction. The oscillations were recorded by a high speed video recorder, and compared with the patterns generated by theoretical calculations.

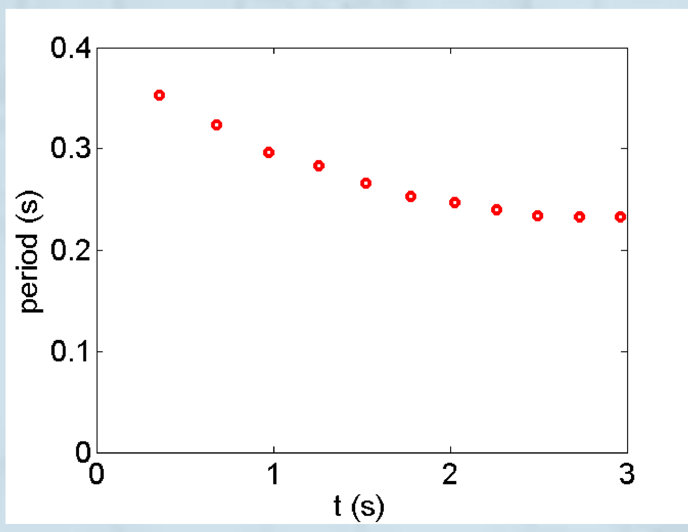
Large Amplitude Confinement





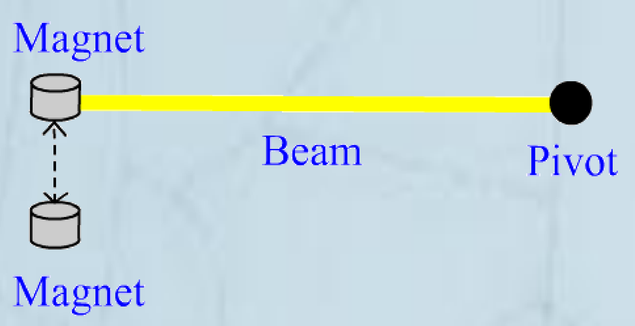
Two confinement with large amplitude and finite damping.

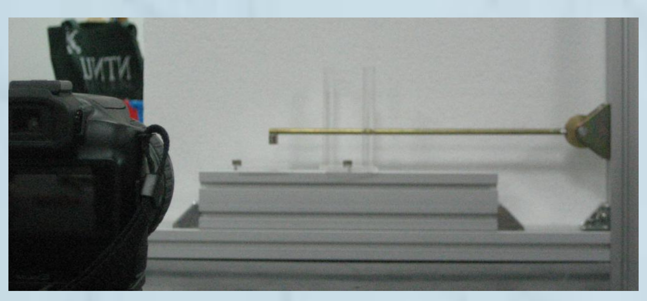




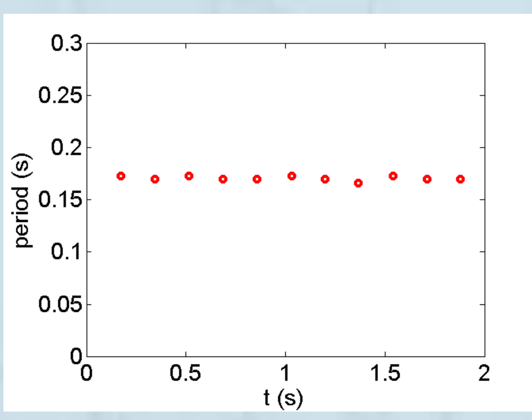
Experimental results. Note that the curve at the bottom turning point is sharper. Also, amplitude decays and period reduces with time.

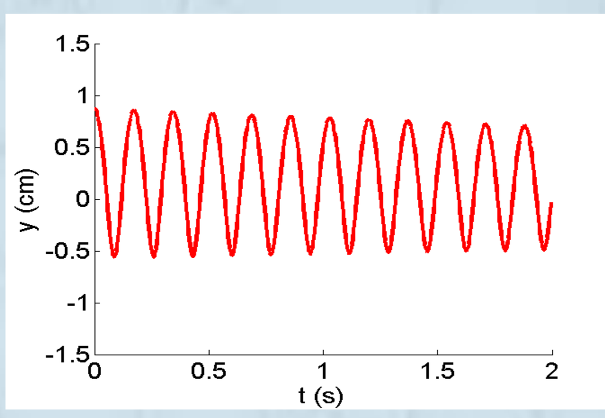
Weak Damping Confinement





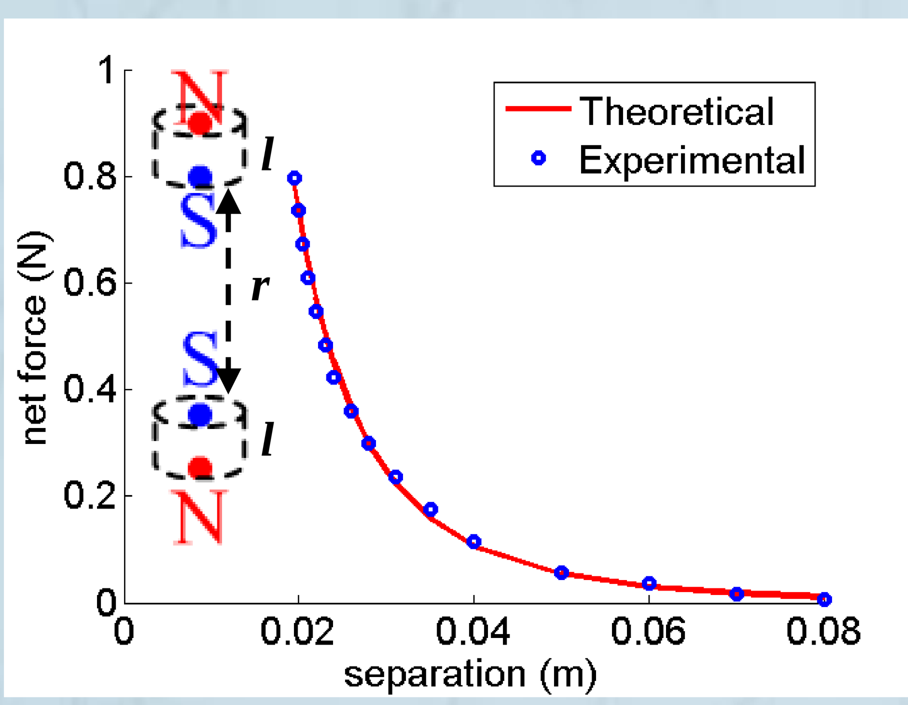
Beam confinement with negligible friction.





Under small amplitude, the oscillation can be considered vertical. The upper magnet exhibits periodic oscillation.

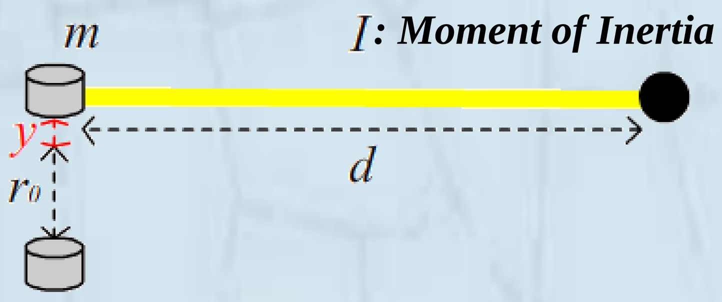
Magnetic Force



$$F_B(r) = C\left(\frac{1}{r^2} + \frac{1}{(r+2l)^2} - \frac{2}{(r+l)^2}\right)$$

Consistency between the magnetic force calculated by magnetic dipole model and measured value.

Equation of Motion



$$\tau = \vec{r} \times \vec{F} = d \times (F_B(r_0+y) - F_B(r_0)) \qquad \tau = I\alpha = I\left(\frac{d^2y}{dt^2}\frac{1}{d}\right)$$

$$\Rightarrow \frac{d^2y}{dt^2} = \frac{d^2}{I}\left(F_B(r_0+y) - F_B(r_0)\right)$$

$$\frac{d^2}{dt^2}\left(\frac{y}{r_0}\right) = \frac{Cd^2}{Ir_0^3}\left((1+\frac{y}{r_0})^{-2} + (1+\frac{y}{r_0}+\frac{2l}{r_0})^{-2} - 2(1+\frac{y}{r_0}+\frac{l}{r_0})^{-2} - 1 - (1+\frac{2l}{r_0})^{-2} + 2(1+\frac{l}{r_0})^{-2}\right)$$

There are only three parameters $\frac{y_0}{r_0}$, $\frac{l}{r_0}$, $\frac{Cd^2}{Ir_0^3}$ in this problem.

For small amplitude oscillation, the force can be linearized.
Oscillation period $T_s = \frac{2\pi}{\sqrt{2\frac{Cd^2}{I}(r_0^{-3} + (r_0+2l)^{-3} - 2(r_0+l)^{-3})}}$

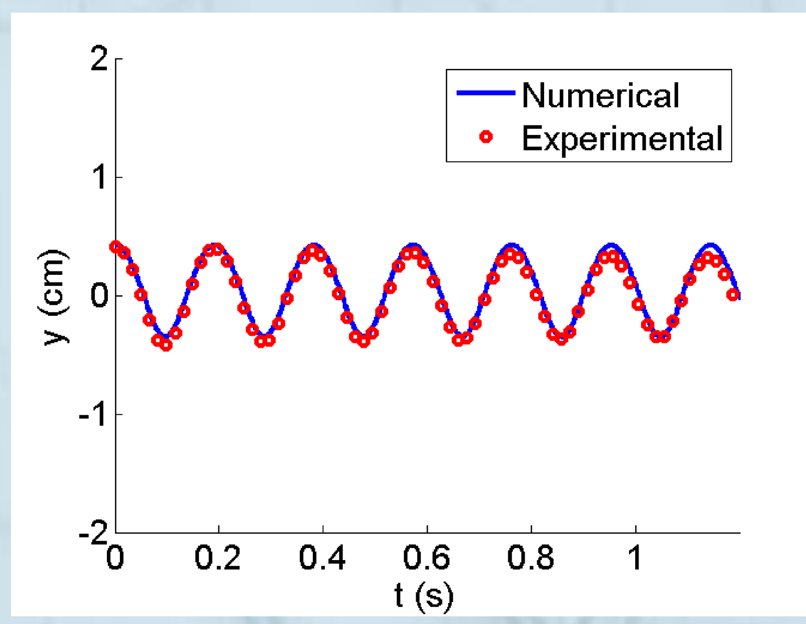
One of the parameters can be written in terms of T_s,

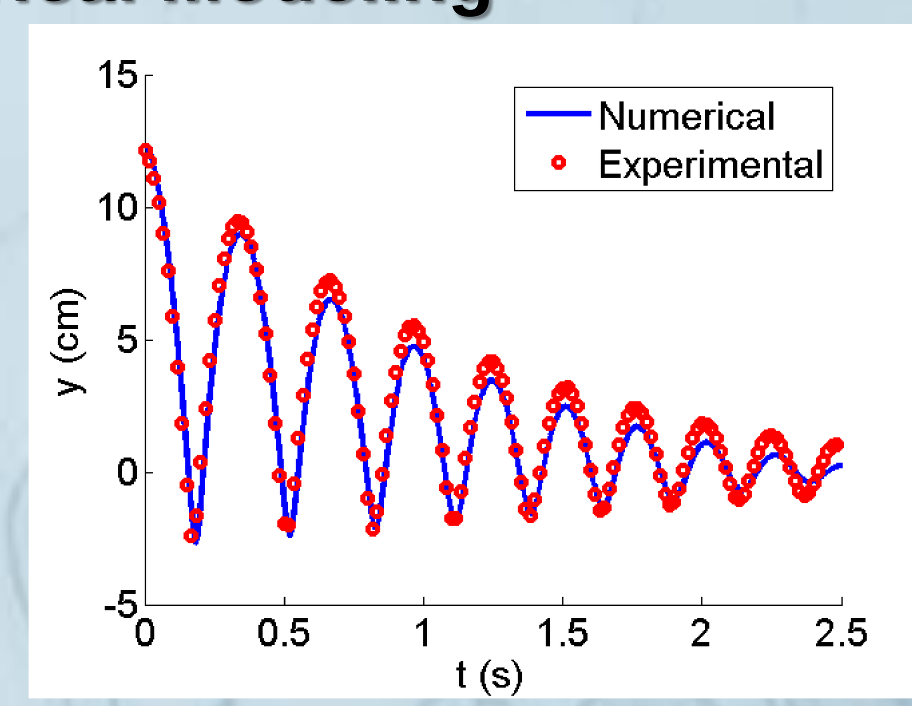
$$\frac{Cd^2}{Ir_0^3} = \frac{2\pi^2}{T_s^2(1 + (1+\frac{2l}{r_0})^{-3} - 2(1+\frac{l}{r_0})^{-3})}$$

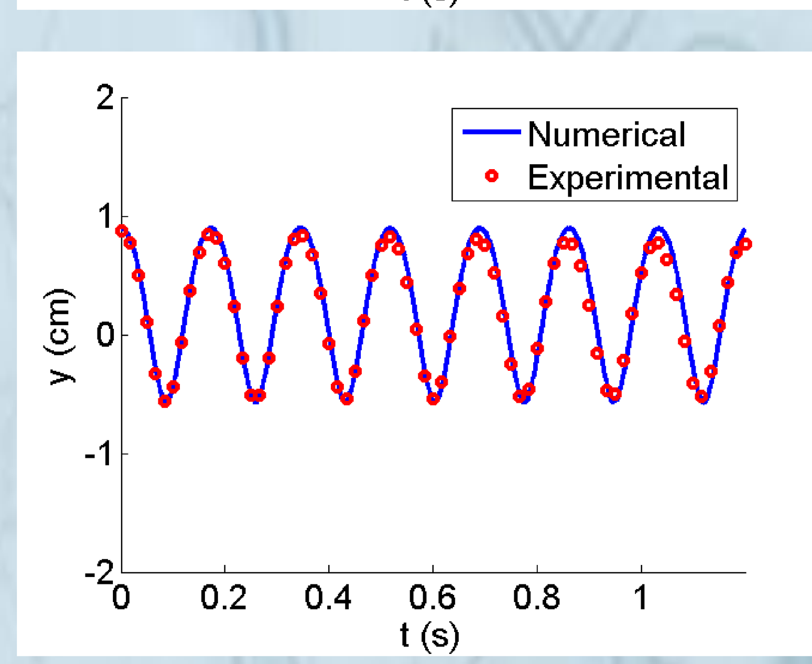
In finite damping case, zeroth and first order term of resistive force are considered.

$$\frac{d^2y}{dt^2} = \frac{d^2}{I}\left(F_B(r_0+y) - F_B(r_0) - b\frac{dy}{dt} \pm f\right)$$

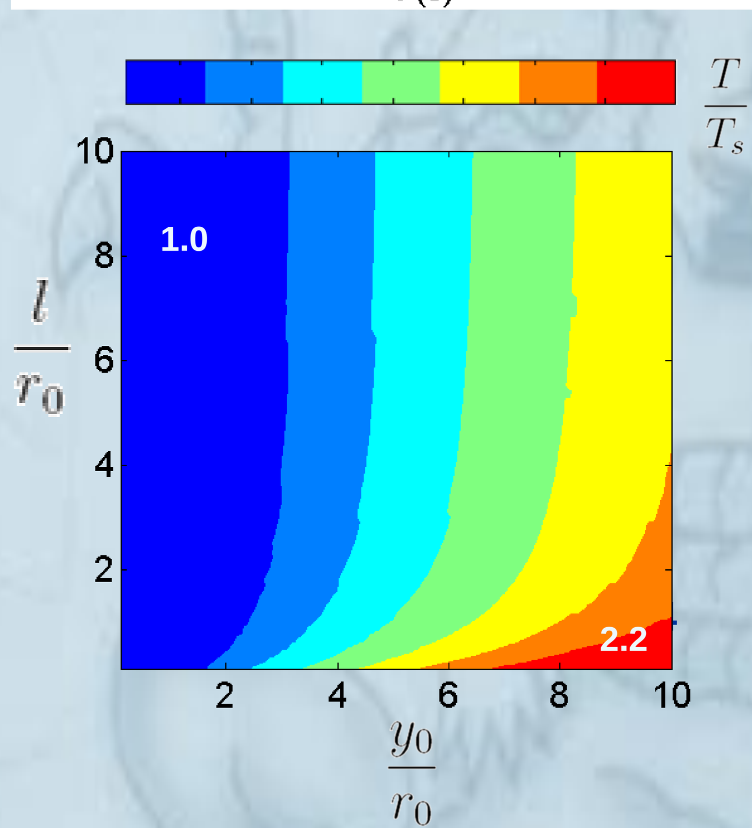
Numerical Modeling



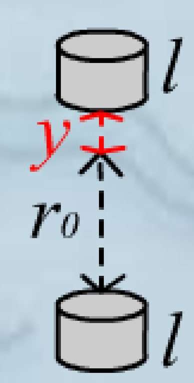




Comparison between numerical result and experimental result in weak damping cases (left) and finite damping case (right).



□ y_0 ↑ · T↑
□ $y_0 \to 0$ · $T \to T_s$
□ $l \to$ large · T X l



Comprehensive solution of $\frac{T}{T_s} = f(\frac{y_0}{r_0}, \frac{l}{r_0})$

Summary

- Three confinements with different purpose are designed.
- Numerical results fit the practical experimental results.
- The results are summarized by period ratio, rescaled amplitude, and rescaled magnet length.