23rd IYPT Problem 13: Shrieking Rod

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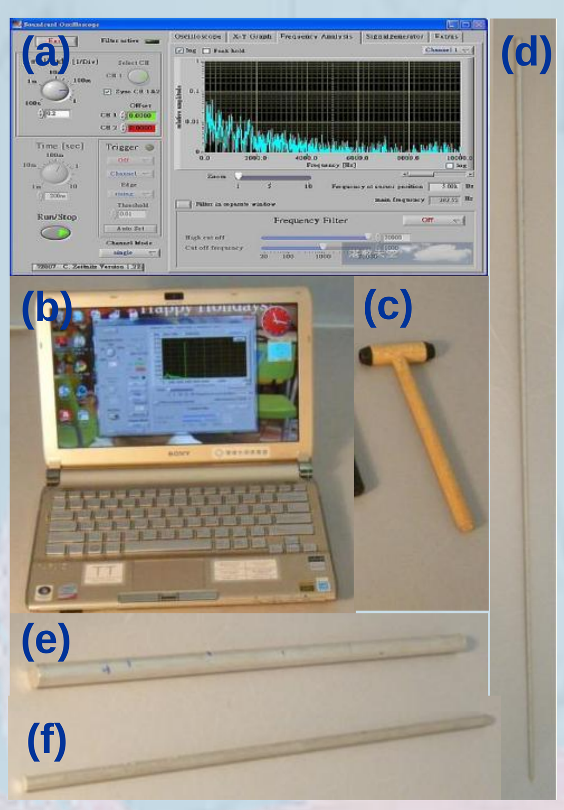
Abstract

A Aluminum rod is held between two fingers and hit. Investigate how the sound produced depends on the position of holding and hitting the rod?

There are two important points needed to be specified, one is the position of holding the rod and the other is the hitting position. We expect the longitudinal wave and the flexural wave will be created and can be observed. However, dependence of the holding position, the modes observed will be different. On the other hand, the hitting position certainly has an effect on the intensities of the created modes, especially the relative intensity of the longitudinal mode and the flexural mode.

Experimental Setup



(a)Soundcard Oscilloscope :

(b)Microphone in notebook :

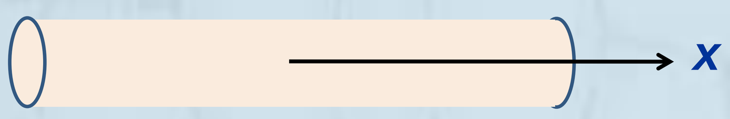
(c)mallet

Aluminum Rods

Rod	Length	diameter
(d)	100cm	6mm
(e)	48cm	19mm
(f)	48cm	9.5mm

Theory

■**Longitudinal Wave Equation :**



$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c_L^2}\frac{\partial^2 \xi}{\partial t^2}$$

C_L :Longitudinal wave speed

E :Young's Modulus

$$c_L = \sqrt{\frac{E}{\rho}}$$

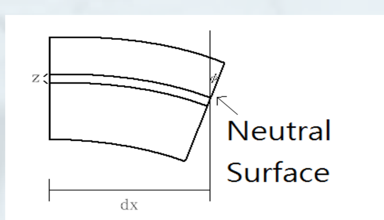
we get $f_{nl} = n\dfrac{c_L}{2L}$ for $n = 1, 2, 3...$

■**Flexural Wave Equation :**

For a cylindrical rod with diameter much smaller than a wavelength.

$$\frac{\partial^4 \eta}{\partial x^4} = -\frac{1}{c_L^2 R_G^2}\frac{\partial^2 \eta}{\partial t^2}$$

$$R_G = \frac{D}{4}$$



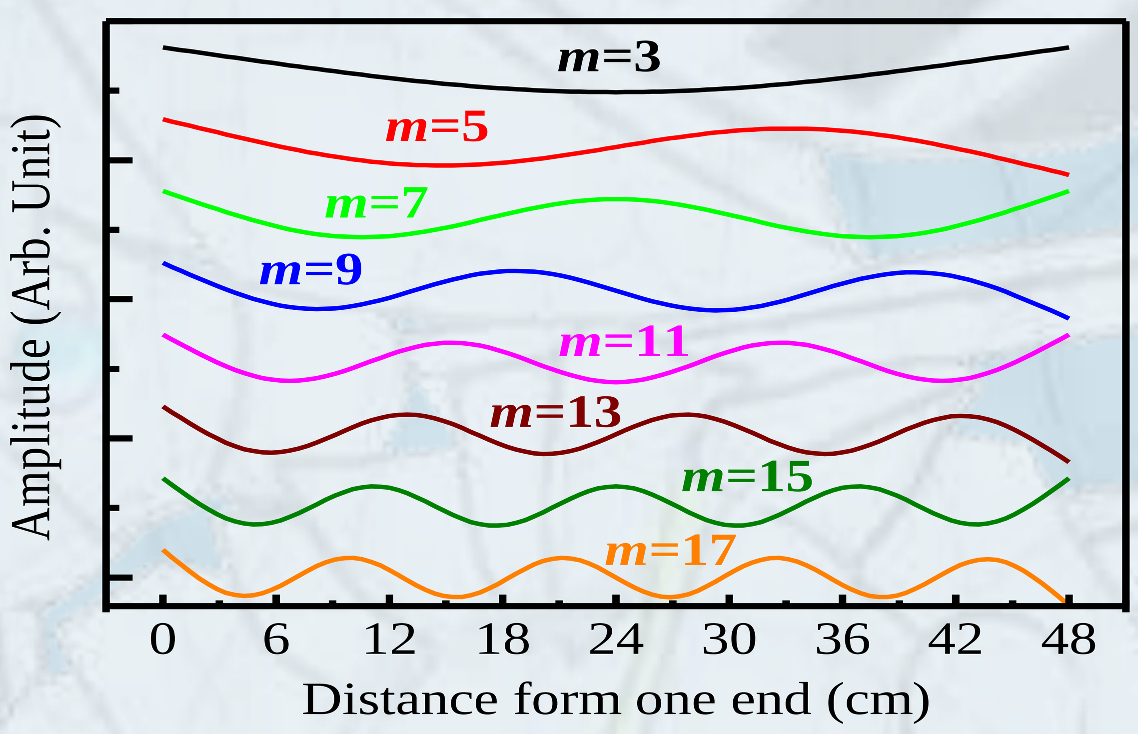
R_G is the radius of gyration of the cross section

$$\eta(x,t) = [\alpha\cosh(kx) + \beta\sinh(kx) + \gamma\cos(kx) + \delta\sin(kx)]\cos(\omega t + \varphi)$$

$$f_m \cong m^2 \frac{\pi c_L R_G}{8L^2}$$

for

m = 3, 5, 7, 9, …

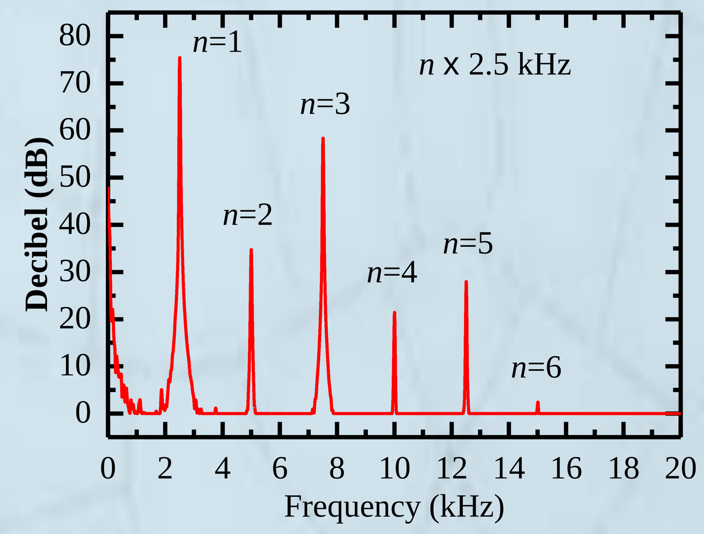


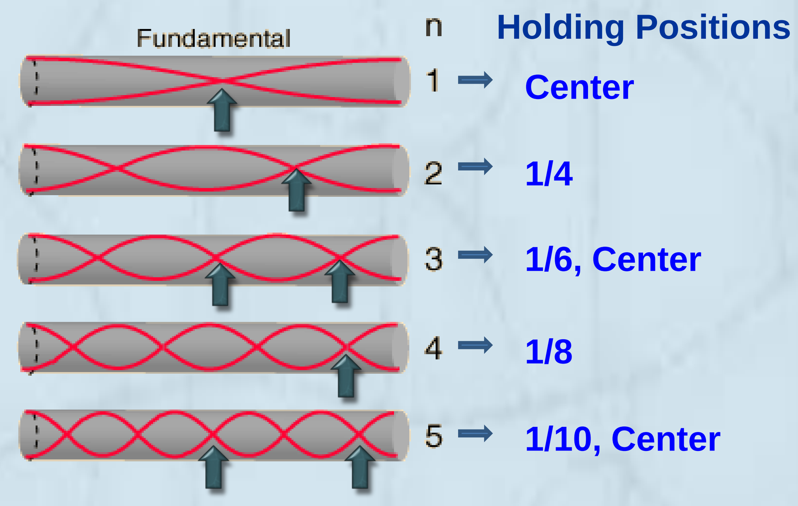
For Aluminum Rod :
E = 69 Gpa, ρ = 2700 kg/m³

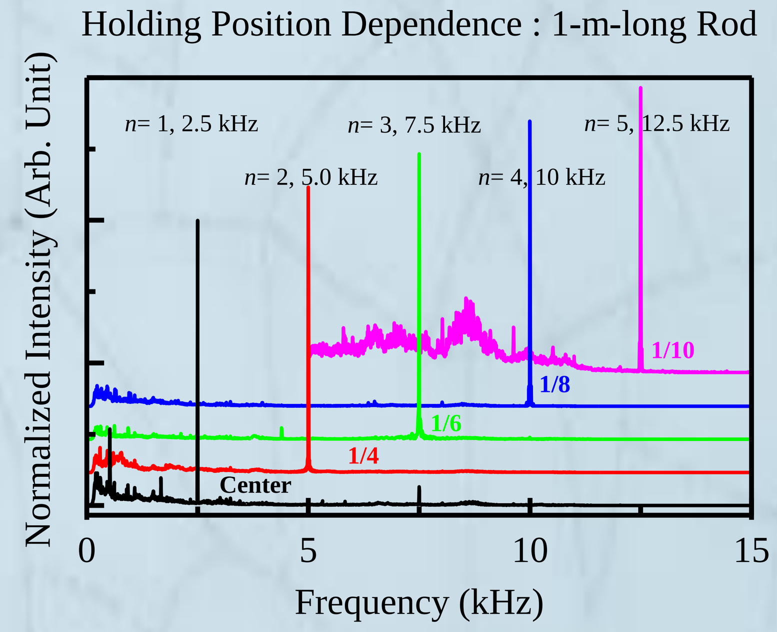
$$c_L \approx 5000 \text{ m/s}$$

Result

■**Longitudinal vibrations of rod(d)**

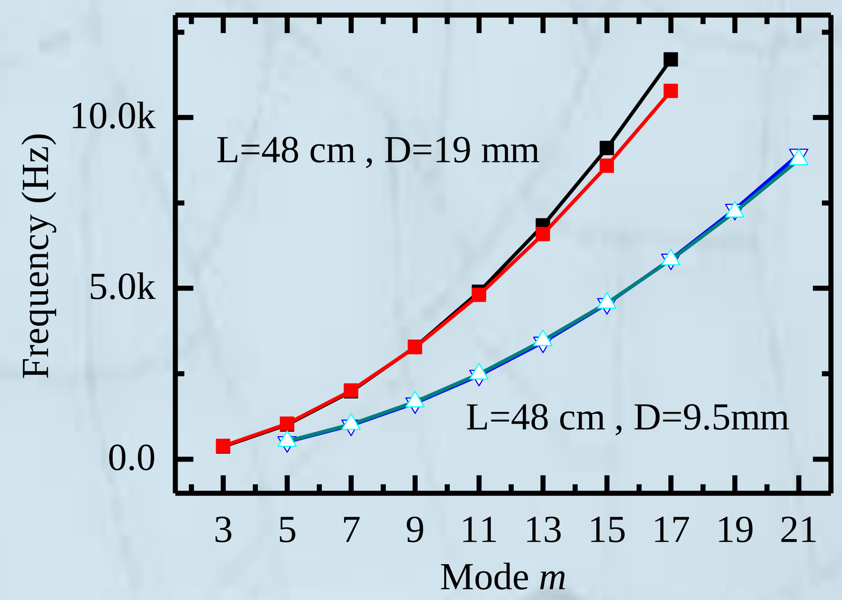






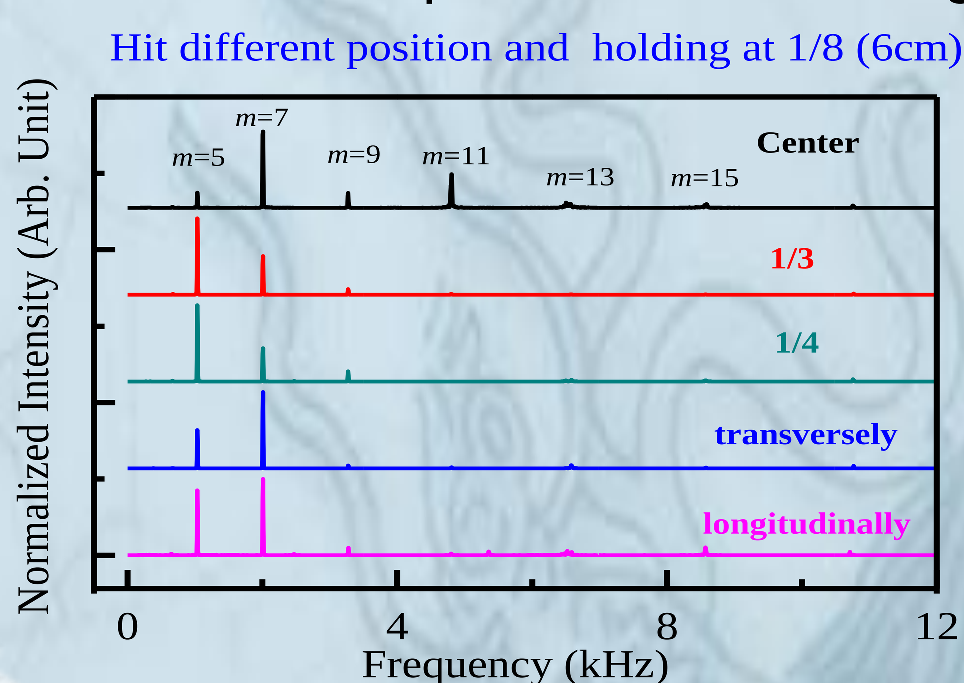
■**Flexural vibration of rod(e) and (f)**

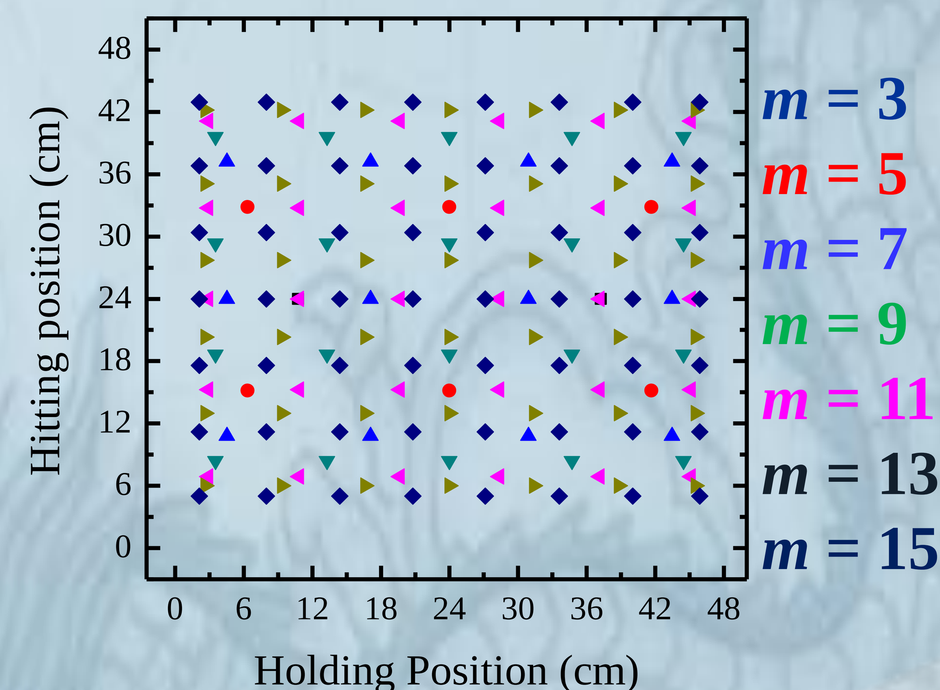
1.Measured and Predicted frequencies of rod(e) and (f)

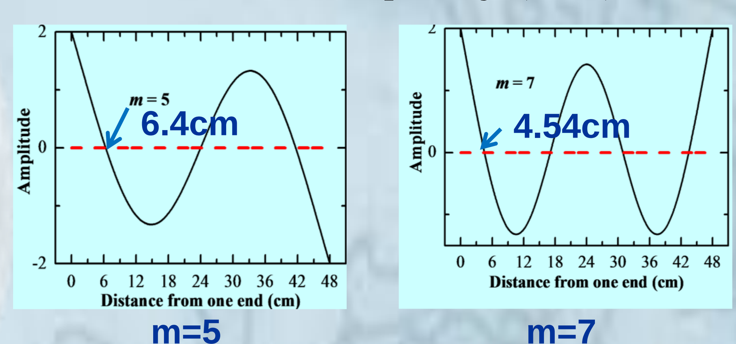


As m increases, frequency also increases, resulting in shorter distance between nodes. Thus, the diameter of the rod becomes more significant.

2.Hit Different position and holding at 1/8(6cm) of rod(e)







Holding the rod at a node while hitting it at an anti-node may enhance the particular mode.

Conclusion

For Longitudinal Mode, we get $f_{nl} = n\dfrac{c_L}{2L}$ for $n = 1, 2, 3...$

For Flexural Mode, by solving the wave equation, we can get the phase diagram of holding and hitting positions for the frequency from 20 up to 20000 Hz.

Reference

[1]Francesca M. F. Mascarenhas(1998)"Hearing the shape of a rod by the sound of its collision". Am. J. Phys., 66, Page #