23th IYPT Problem : Liquid Light Guide

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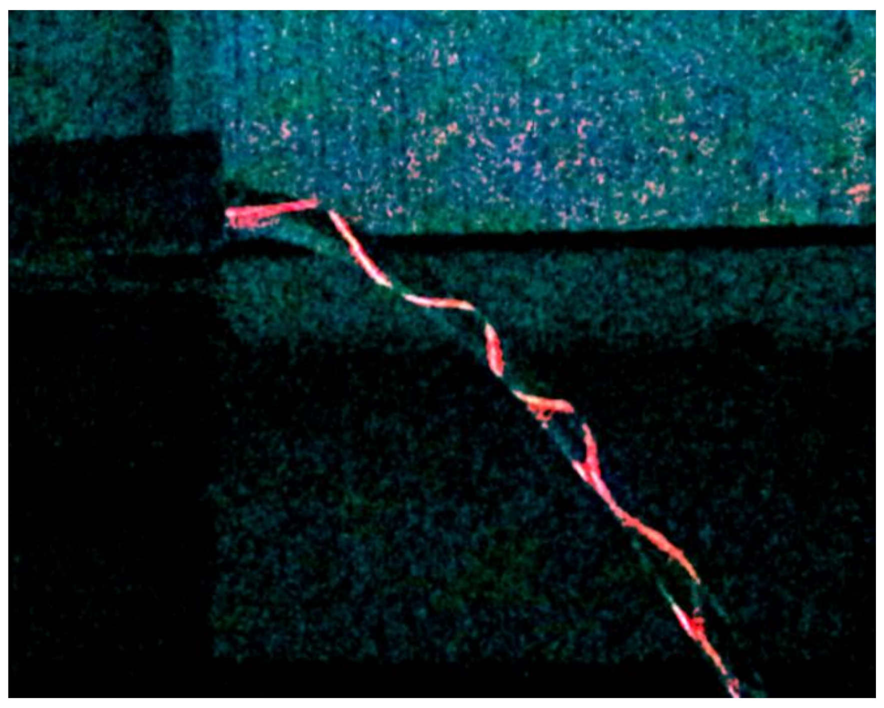
Abstract

A transparent vessel is filled with a liquid (e.g. water). A jet flows out of the vessel. A light source is placed so that a horizontal beam enters the liquid jet. Under what conditions does the jet operate like a light guide?

According to theoretical calculation, the liquid jet should act as a light guide until the water level above the vessel center reaches the critical height H which is equal to 0.648d, where d means diameter of the vessel hole. The result of experiment is close to that of theoretical calculation. Deformation of the liquid flow, including deformed flow, turbulent flow and flow broken into drops, will lead liquid jet unable to act like a light guide.

Experimental Setup



Instruments:

A 632.8nm He-Ne Laser

A optical platform

A reservoir

A basin

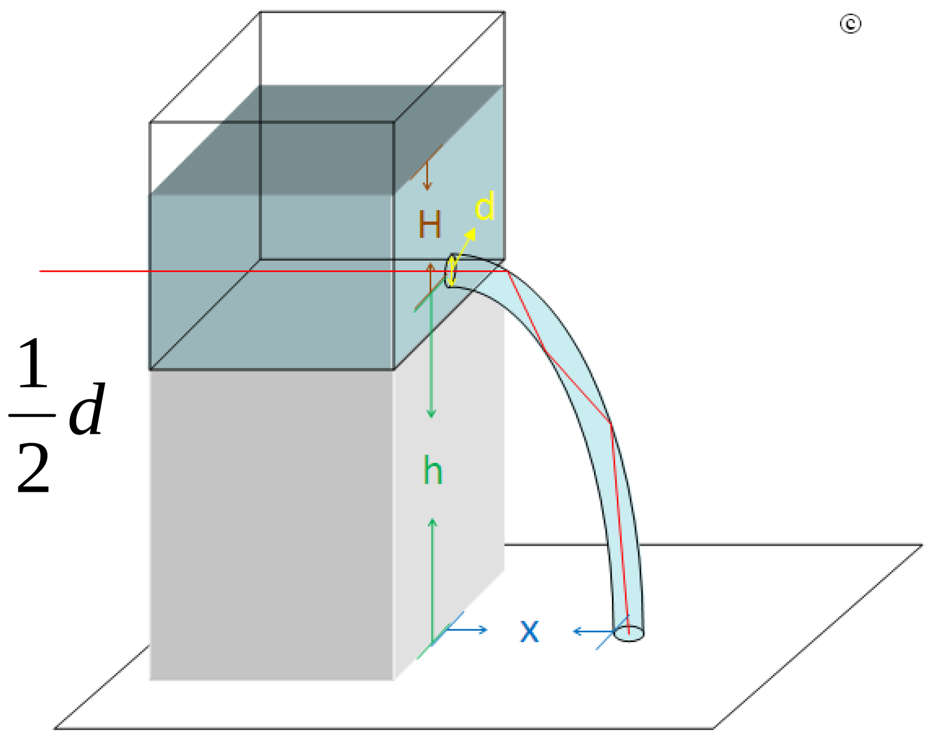
Theory

Path of jet equation:

$$y = -\frac{x^2}{4H} + h + \frac{1}{2}d$$

let $\vec{d}_1 = (1,0)$

Then the reflecting point is P(t,h)



Substituting P into $y = -\frac{x^2}{4H} + h + \frac{1}{2}d$

we can write $h = -\frac{t^2}{4H} + h + \frac{1}{2}d$

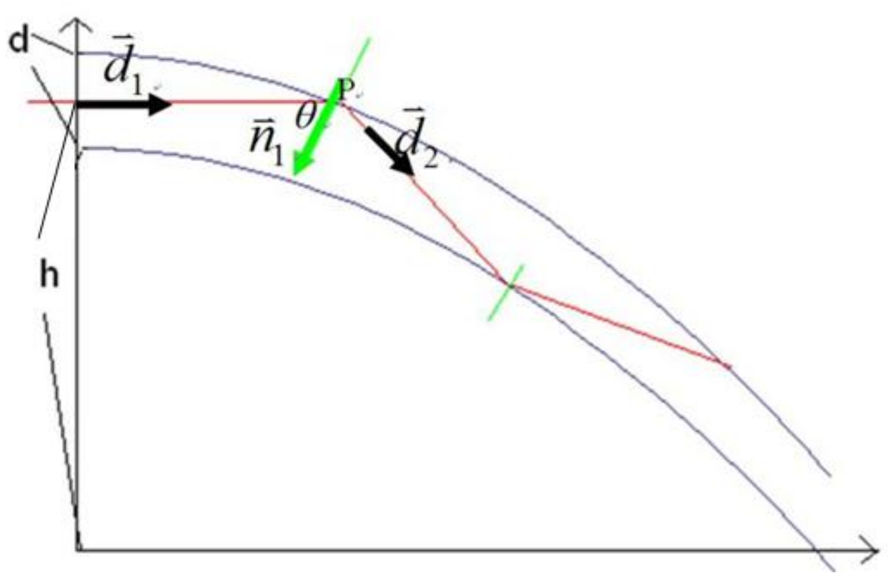
$$\Rightarrow t = \sqrt{2dH} \Rightarrow P(\sqrt{2dH}, h)$$

To get the tangential line at a given point, it's slope is:

$$\frac{dy}{dx} = -\frac{x}{2H}$$

Tangential line is:

$$T_1 : y = -\frac{\sqrt{2dH}}{2H}x + h + d$$



Normal vector can be written: $\vec{n}_1 = (\sqrt{2dH}, 2H)$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{d}_1}{|\vec{n}_1||\vec{d}_1|}$$

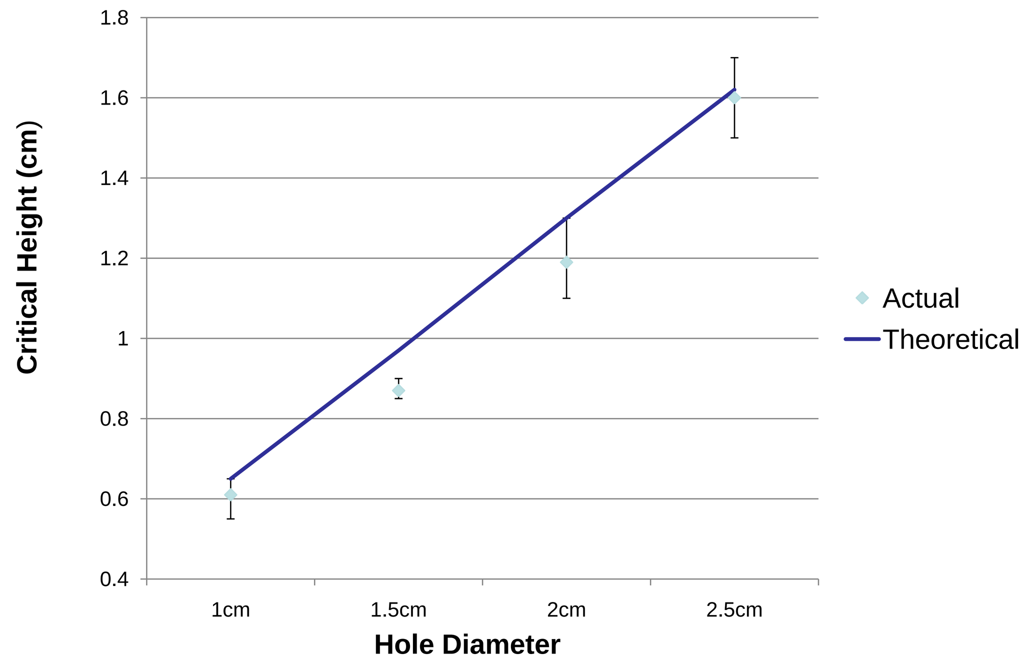
According to Snell's law , The critical angle for water is:

$$\sin^{-1}\left(\frac{n_2}{n_1}\right) \cong 48.8°$$

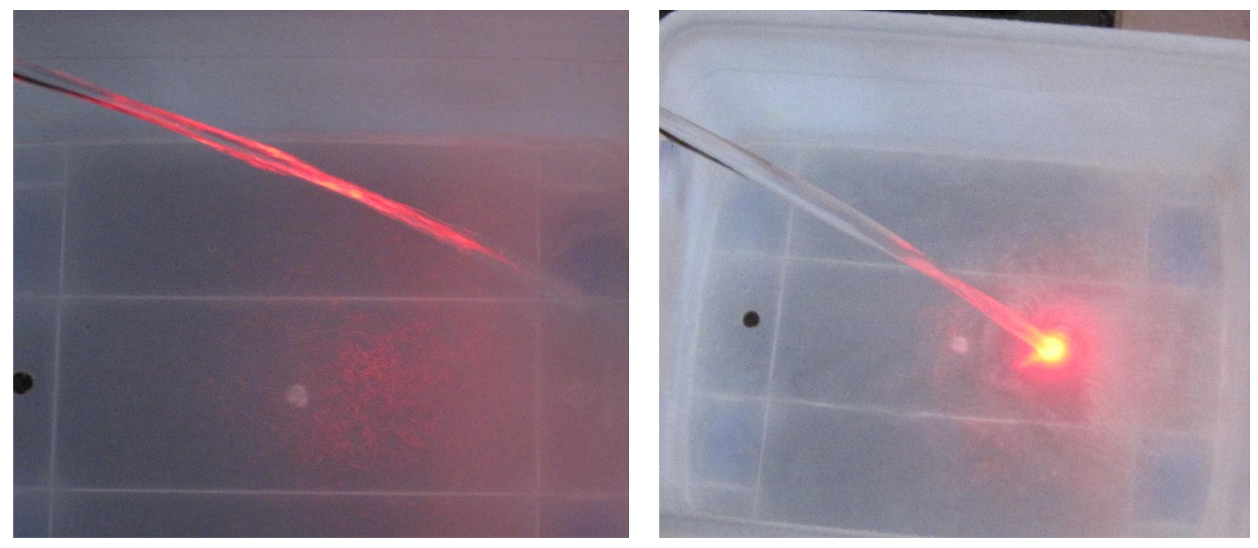
When the stream acts as a light guide, the incident angle must be greater than the critical angle, Thus,

$$0 < \frac{\sqrt{2dH}}{\sqrt{2dH + 4H^2}} < \cos 48.8° \Rightarrow H > \frac{(1-0.66^2)\cdot 2d}{4\cdot 0.66^2} \Rightarrow H > 0.648d$$

Result



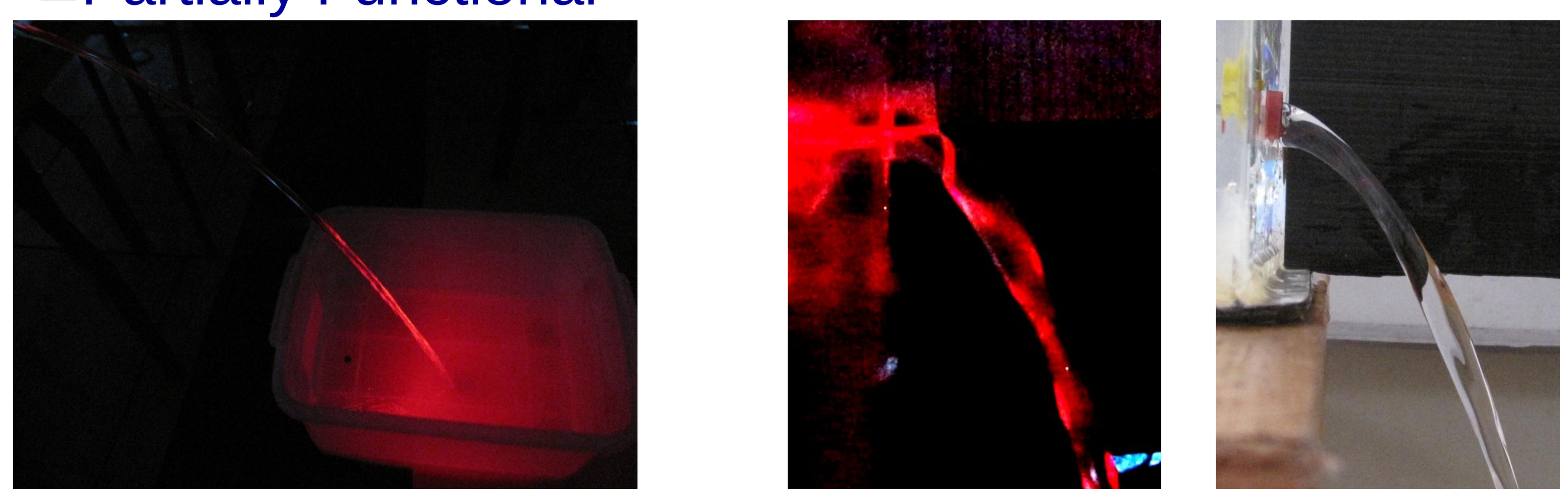
In the theory, the jet should act as a light guide until the water reaches the critical height. However, various other effects that causes the light guide to fail are observed.



Fail vs. Success

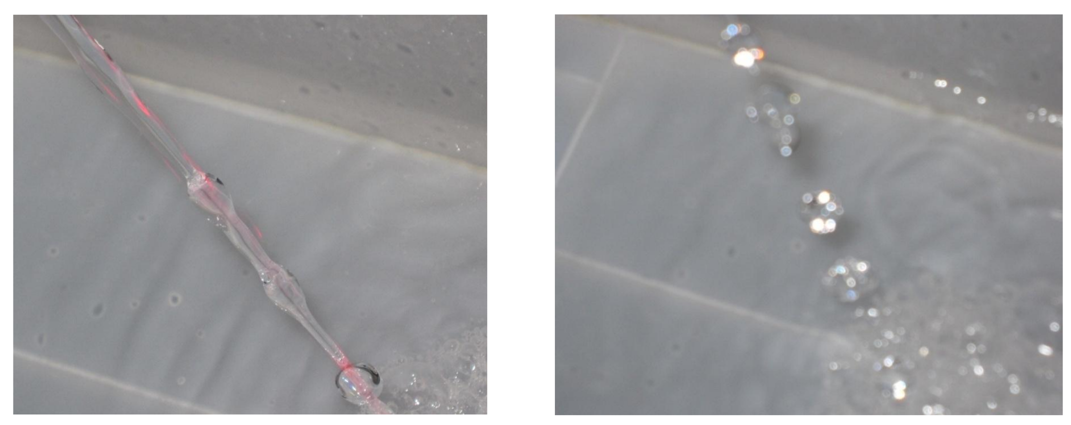
Other Influential Factors:

■Partially Functional



Diameter: 1cm
Water Height: 5cm

Diameter	1cm	1.5cm	2cm	2.5cm
Deformed Flow	<2.5cm	<3cm	<2.5cm	<2.5cm



Diameter	1cm	1.5cm	2cm	2.5cm
Deformed End	<6cm	<4cm	<3.5cm	<3cm
Drops	<3.5cm	N/A	N/A	N/A

Diameter / Effect	1cm	1.5cm	2cm	2.5cm
Success	>6cm	>4cm	>3.5cm	>3cm
Deformed End	<6cm	<4cm	<3.5cm	<3cm
Drops	<3.5cm	N/A	N/A	N/A
Deformed Flow	<2.5cm	<3cm	<2.5cm	<2.5cm
No Reflection	<0.61cm	<0.87cm	<1.2cm	<1.6cm

Conclusion

The jet will act as a light guide except in the following cases:

1. Incident angle < Critical angle
   - Speed of jet is too slow
   - Surface becomes too turbulent
   - Jet becomes deformed
2. The flow breaks into drops

Reference

[1] *S. P. Lin , R. D. Reitz*
Annu. Rev. Fluid Mech. 1998. 30:85–105