23rd IYPT Problem : Grid

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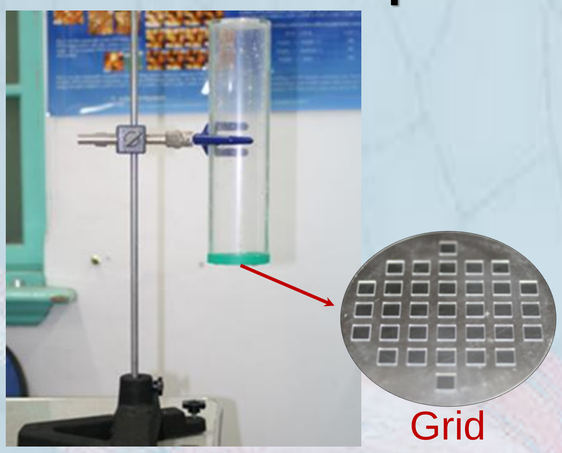
Abstract

A plastic grid covers the open end of a cylindrical vessel containing water. The grid is covered and the vessel is turned upside down. What is the maximal size of holes in the grid so that water does not flow out when the cover is removed?

When water lies above air in a gravitational field, a small perturbation at the interface will increase the local pressure difference, which will in turn enhance the motion of the interface until the water flows out. This is known as the Rayleigh-Taylor instability. However, the surface tension can provide a restoring force to hold the water. Both surface tension force and gravitational force are affected by the shape of the water surface, so the competition between two forces will be determined by the geometrically preferred water wave. In this study, we discussed the effect of several parameters, including arrangement of the grid, number of holes, spacing between the holes, thickness of the grid, and the water height.

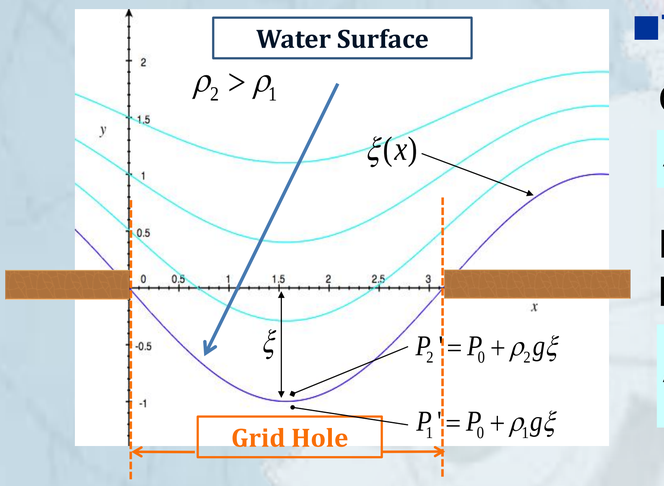
Experimental Setup



(a) Cylindrical Vessel
(b) Grid
1. Hole Size
2. Hole Arrangement
3. Hole Spacing
4. Thickness

(c) Metal Holder
(d) Plastic Cover

Theory



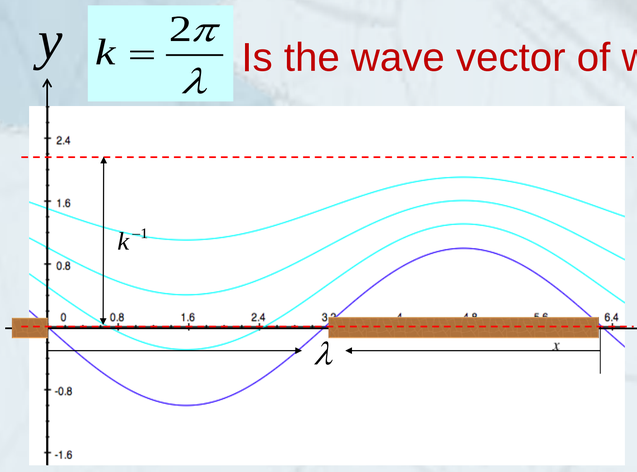
■**Two competing forces:**

Gravitational Force F_g

$\Delta P_g = \Delta\rho g\xi$

Restoring Force caused by Surface Tension F_{ST}

$\Delta P_{ST} = \frac{\sigma}{R_c}$



$k = \frac{2\pi}{\lambda}$ Is the wave vector of water surface modes

Surface modes that decay by $\exp(-ky)$ are formed

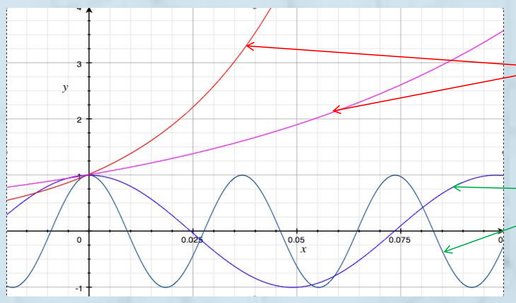
The effective distance of the disturbance is k^{-1}

The total effective mass is $m = m_1 + m_2 = \rho_1 \frac{A}{k} + \rho_2 \frac{A}{k}$

■**Motion Formula of Water Surface:** $m\ddot{\xi} = A\xi(\Delta\rho g - k^2\sigma)$

From $m\ddot{\xi} = A\xi(\Delta\rho g - k^2\sigma)$, $m = m_1 + m_2 = \rho_1 \frac{A}{k} + \rho_2 \frac{A}{k}$ We get $\ddot{\xi} = \frac{k\xi(\Delta\rho g - k^2\sigma)}{\rho_1 + \rho_2}$

$\xi \propto \exp(\gamma t)$ $\gamma = \sqrt{\frac{k\xi(\Delta\rho g - k^2\sigma)}{\rho_1 + \rho_2}}$ $\Rightarrow k_c = \sqrt{\frac{\Delta\rho g}{\sigma}}$

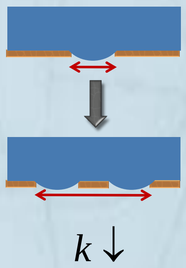


If $k > k_c$, $\xi(t)$ will be a sinusoidal oscillation, water stays in grid
If $k < k_c$, $\xi(t)$ will be a exponential growth, water flows out

Result

1. Changing Arrangement of Holes

Hole Array	1x6	1x5	1x4	1x1
Maximum Size	7^2mm^2	8^2mm^2	9^2mm^2	13^2mm^2

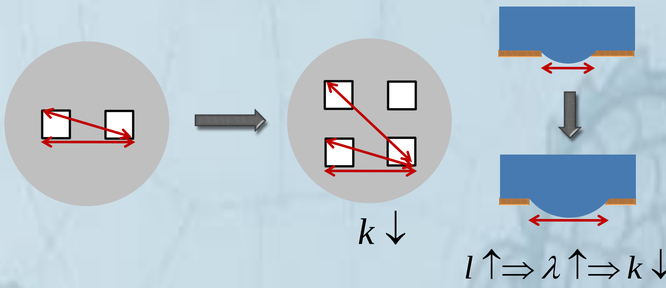


$k \downarrow$

2. 2-Dimensional Arrangement & Different Hole Sizes

Hole Size	4^2mm^2	5^2mm^2	6^2mm^2
Result	Success		

Hole Size	7^2mm^2	7.5^2mm^2	8^2mm^2	9^2mm^2
Result	Success Below 4x4	Success Below 3x3	Success Below 2x2	Fail

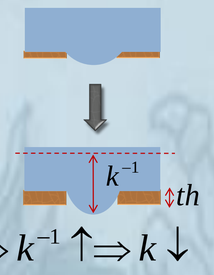


$k \downarrow$

$l \uparrow \Rightarrow \lambda \uparrow \Rightarrow k \downarrow$

3. Changing Thickness of Grid

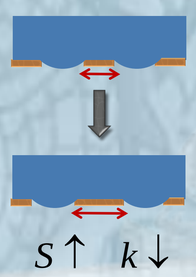
Hole Size	7^2mm^2		
Thickness	1mm	2mm	3mm
Result	Success Below 4x4	Success Below 3x3	Fail



$th \uparrow \Rightarrow k^{-1} \uparrow \Rightarrow k \downarrow$

4. Changing Spacing Between Holes

Hole Size	7^2mm^2		
Spacing	4mm	7mm	10mm
Result	Success Below 4x4	Success Below 3x5	Success Below 3x4



$S \uparrow \quad k \downarrow$

Conclusion

The main parameters to affect the maximum hole size are:
- The number of holes
- The dimension of arrays
- The spacing between holes

Secondary parameters include:
- The diameter of the vessel (when near λ_c=17 mm)
- The thickness of the grid (when near $1/k_c$=3 mm)

Irrelevant parameter is the height of water

Reference

[1] Piriz, A. R., Cortazar, O. D., Lopez Cela, J. J., *The Rayleigh-Taylor Instability*