23rd IYPT Problem : Brilliant Pattern

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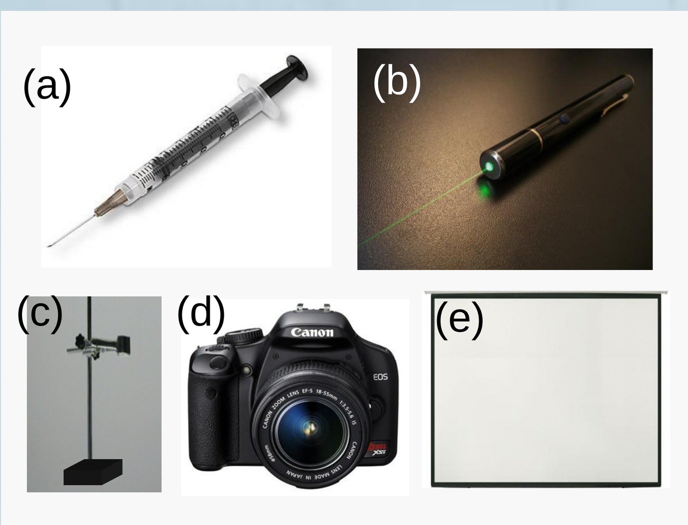
Abstract

Suspend a water drop at the lower end of a vertical pipe. Illuminate the drop using a laser pointer and observe the pattern created on a screen. Study and explain the structure of the pattern.

We consider the laser beam with geometric and wave characteristics. The experimental result reveals that the brilliant patterns are demonstrated by reflection, refraction, and diffraction in the simple system. Furthermore, the analytical analyses have a good agreement with the experimental results.

Experimental Setup



(a) Syringe with water

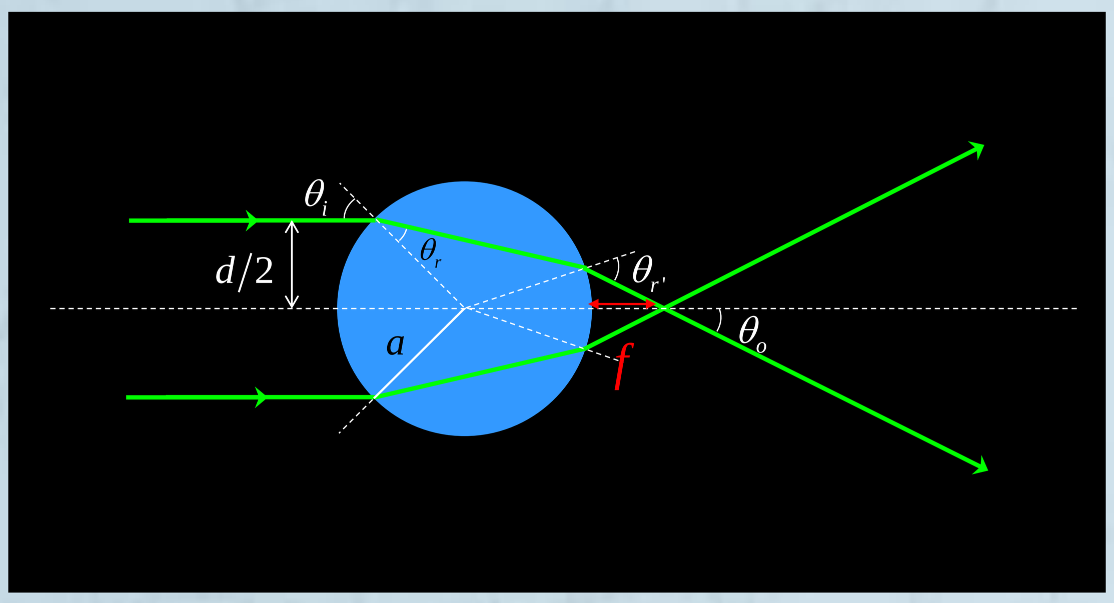
(b) Laser pointer (c) Holder

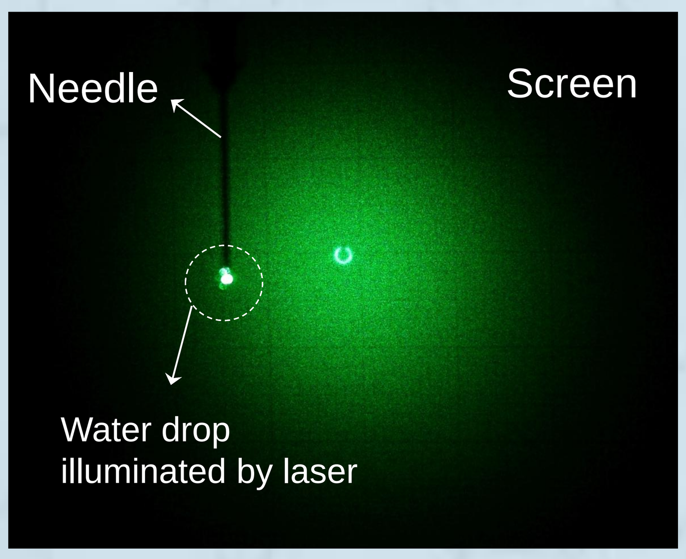
(d) Camera (e) Screen

Theory and Experimental results

■ Geometric Optics (Refraction) :

Laser beam passes through the water drop and focus with the focal length f ; the water drop acts as a magnifier.



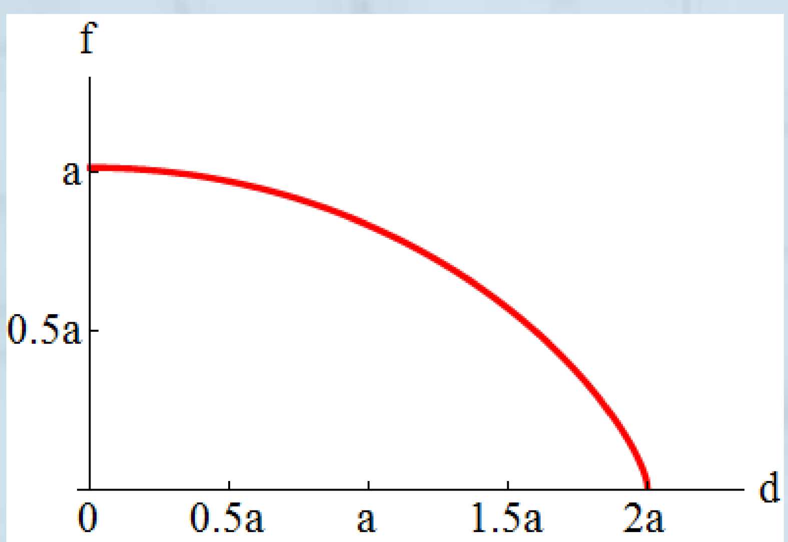


d :diameter of laser beam

a :radius of water drop

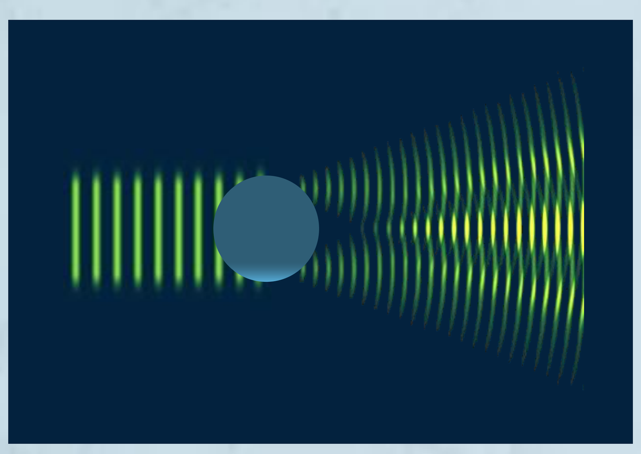
$\sin(\theta_i) = d/2a$ $\quad d\uparrow \Rightarrow f\downarrow$

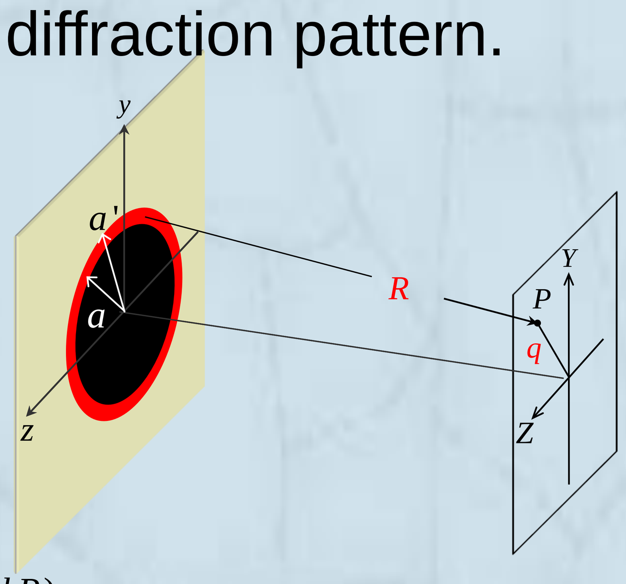
$$f = a\left[\cos(2\theta_r - \theta_i) + \frac{\sin(2\theta_r - \theta_i)}{\tan(2\theta_i - 2\theta_r)} - 1\right]$$



■ Wave Optics (Diffraction) :

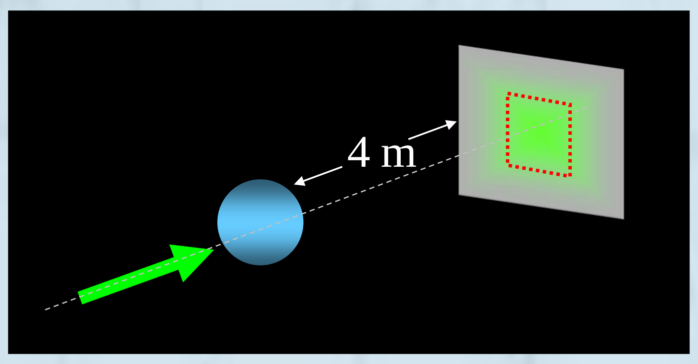
Using Huygens's principle, we take the points outside the obstacle (water drop) as the source of spherical secondary wavelets and sum all of them to reconstruct the diffraction pattern.



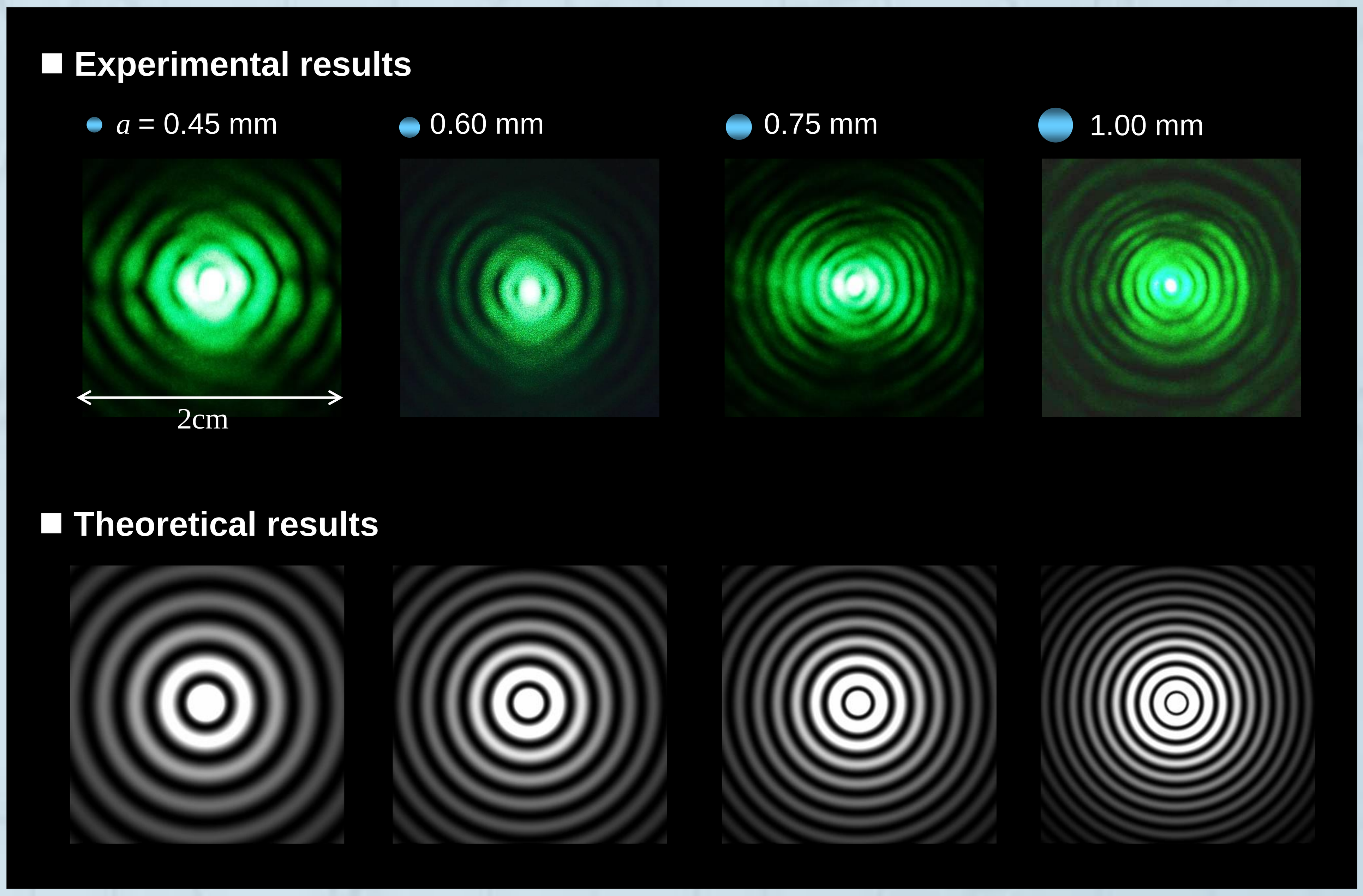


$$dE = \frac{\varepsilon_A}{r} e^{i(\omega t - kr)} dS \quad \Rightarrow E = \frac{\varepsilon_A e^{i(\omega t - kR)}}{R} \iint_{\text{out of aperture}} e^{ik(Yy+Zz)/R} dS$$

$$E \cong \frac{\varepsilon_A e^{i(\omega t - kR)}}{R} 2\pi\,(a' - a) J_0\left(\frac{kaq}{R}\right) \quad I = \frac{2\varepsilon_A^2 \pi (a' - a)^2}{R^2} \left[J_0(kaq/R)\right]^2$$



Different sizes of water drops are used to observe the diffraction patterns on the screen far from water drop with 4 meters. We fix the region of 2x2 cm^2 on the screen to compare the difference. The water drop is larger, the interference fringe is denser.



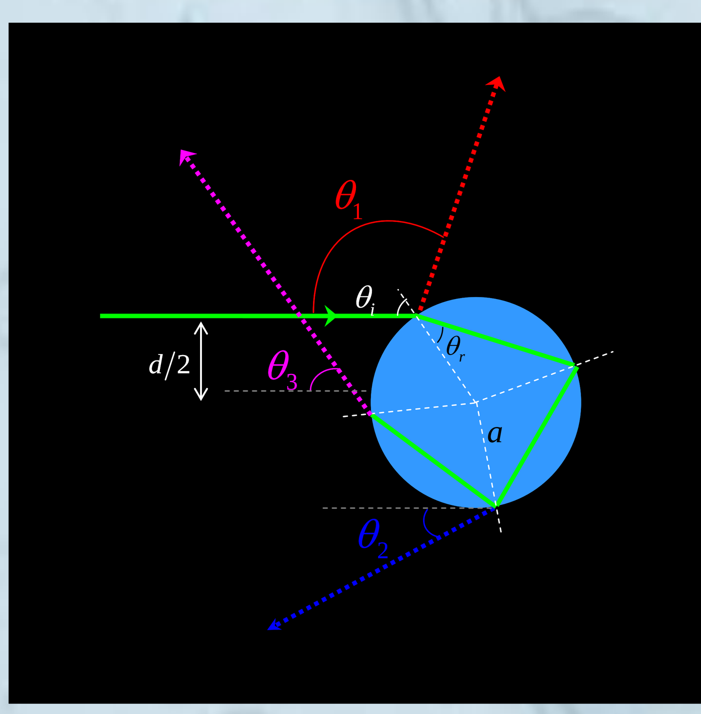
■ Geometric and Wave Optics (reflection, refraction, interference) :

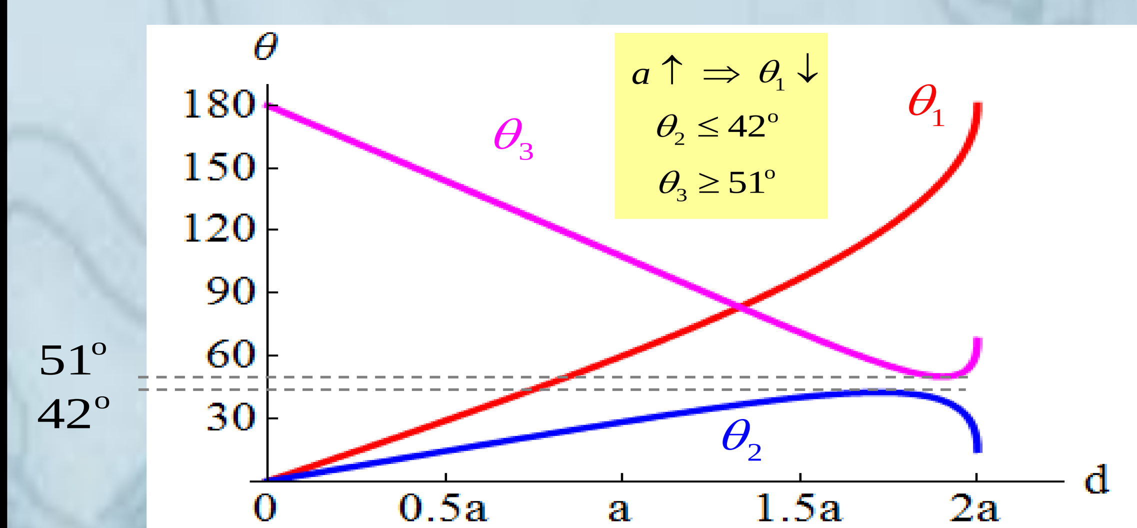
We consider the laser beam as three parts

Part (I): Reflect directly $\Rightarrow \theta_1 = 2\theta_i$

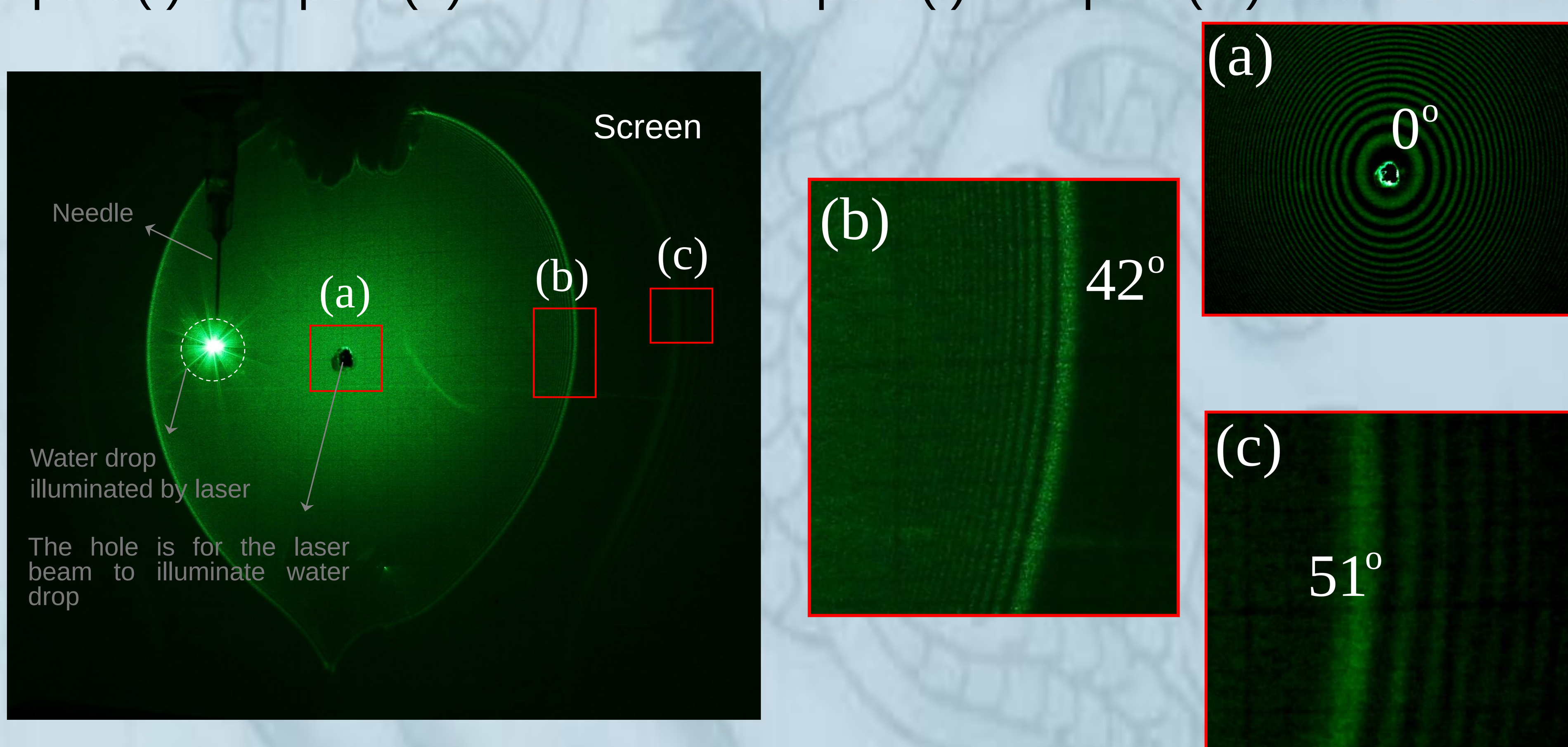
Part (II): Two refractions and one reflection $\Rightarrow \theta_2 = 4\theta_r - 2\theta_i$

Part (III): Two refractions and two reflections $\Rightarrow \theta_3 = 180^o + 2\theta_i - 6\theta_r$





The interference patterns are resulted from the superposition of part (I) and part (II) within 42^0 and part (I) and part (III) outside 51^0.



Reference

[1] Optics (4th Edition) by Eugene Hecht (2001).