The singing saw

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Problem Nr. 2: Singing Saw

Some people can play on a handsaw. How do they get different pitches? Give a quantitative description of the phenomenon.

How to play the singing saw:

- The handle of the saw is clamped in a special holding facility on the bottom
- The left knee touches the saw blade in order to stabilize the instrument.

How to play the singing saw:

- With the help of a metallic holder the saw is bent in form of a s
- One uses a normal violin bow to play the saw
- Through vibrating with the left hand one can also cause beautiful vibrato

First experiments:

Singing saws in general:

- You can create a sound with nearly all normal saws, but it is quite difficult and does not always sound very harmonic.
- The teeth of a saw are not significant as far as they are not crossed. In this case the sound would be damped.

Our Singing saw

For our further experiments we used a special musical saw

Our Singing saw

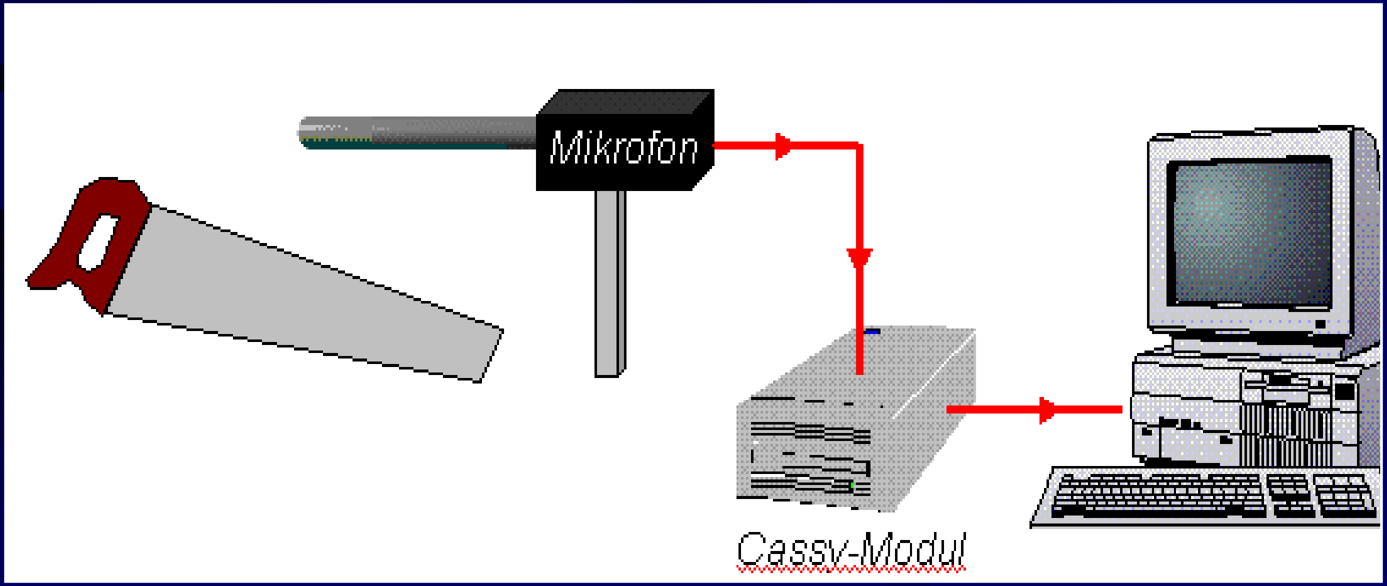
Data of the saw

- Length: 107.7 cm
- Largest blade width 17.7 cm
- Thickness 0.85 mm
- Sound spectra: ≈ 350 – 1500 Hz
- Material: tempered, cold rolled steel

Experiments:

1. Recording of tones and Fourier Analysis:

<u>Experimental set-up:</u>

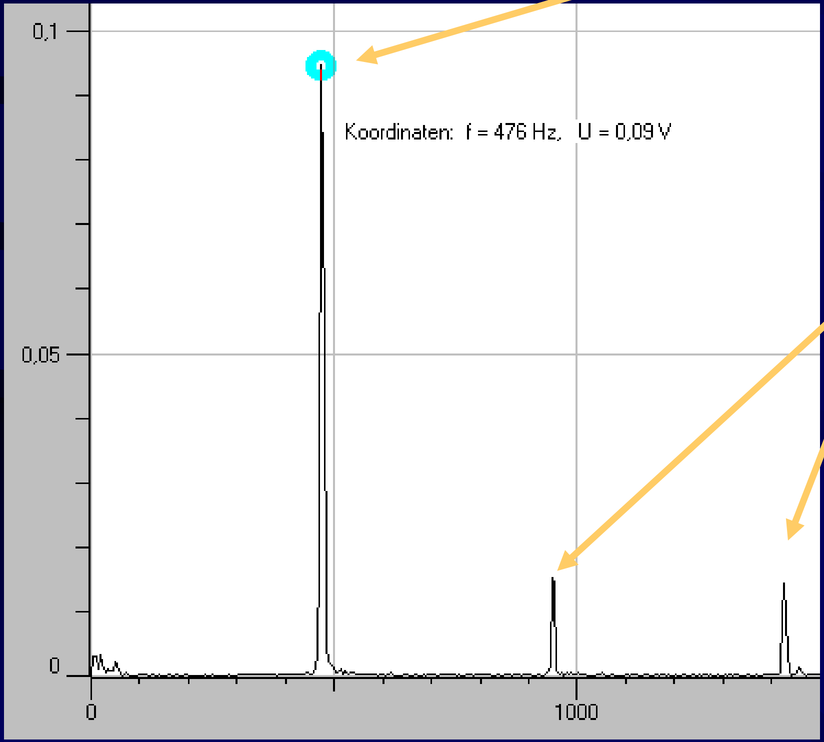


Fourier Analysis:

The Fourier Analysis is a mathematical method which determines the different frequencies of the tone and their intensities and plots them in a diagram.

Voltage in V:

Basic frequency 476 Hz



Frequencies of the overtones

Frequency in Hz:

2. Chladni Sound Figures:

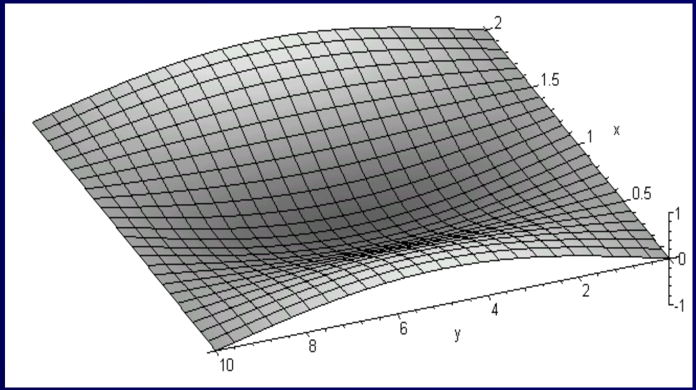
- We spread small pieces of iron on the saw blade and then played a tone

You can see the

We observed

Evaluation of the Chladni Sound Figures:

- Only a part of the saw oscillates
- Boundary conditions:
  - In X- direction:
    free edges, 2 nodelines
  - In Y- direction:
    clamped edges



- The oscillation consists of transversal waves

3. Determination of the speed of sound in the saw blade:

- We have standing transversal waves in our saw blade
- But it was not possible to measure the speed of sound of them experimentally

From the formulas of transversal and longitudinal waves we know:

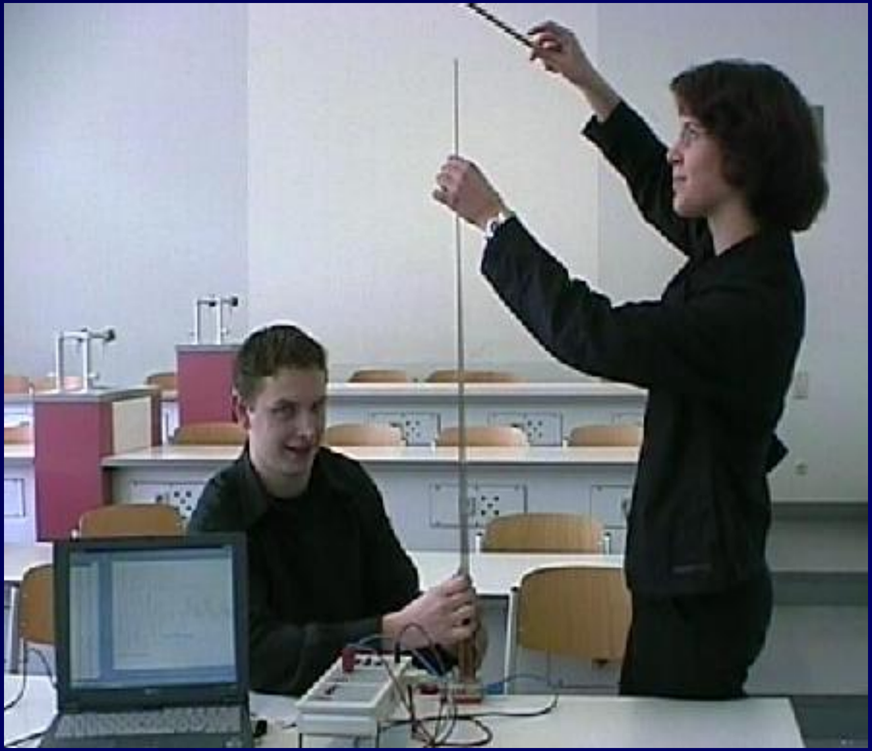
$$\frac{c_t}{c_l} = .59$$

We could measure the speed of sound of longitudinal waves:

Experimental set-up:

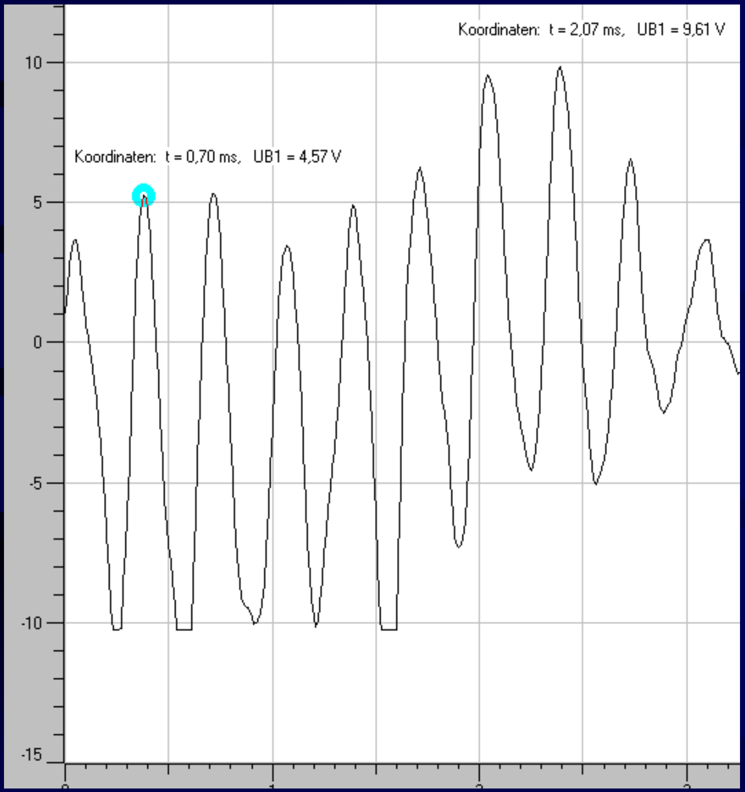
The saw lies on a piezoelectric crystal.

With a metallic stick we give a compression pulse on the long side of the saw blade.



Analysis:

Voltage in V:



Time in s:

The compression pulse is reflected at both ends of the saw blade:

$$c = 2s/T$$

c: Speed of sound

s: Length of the saw blade

T: Time between the maxima

Speed of sound of longitudinal waves:

- For a straight saw blade we measured an average speed of sound of $c_l = 5693$ m/s
- Speed of sound of tranversal waves:

$c_t = 0.59 c_l = 3353$ m/s

$\Rightarrow$ This corresponds well to our calculated value of c_t which is:

$$c_t = \sqrt{\frac{E}{2(1+\nu)\rho}} = 3120\, m/s$$

E: Young‘s modul of elasticity
ρ: Density of steel
ν: Poissons ratio (about 0,3)

4. Determination of the vibrating part of the saw

Experimentel set-up:

The saw is clamped into a special holding device.

This device allows us to make reproducible and exact measure-ments of the vibrating part of the saw blade.

Measurement of the vibrating part of the saw

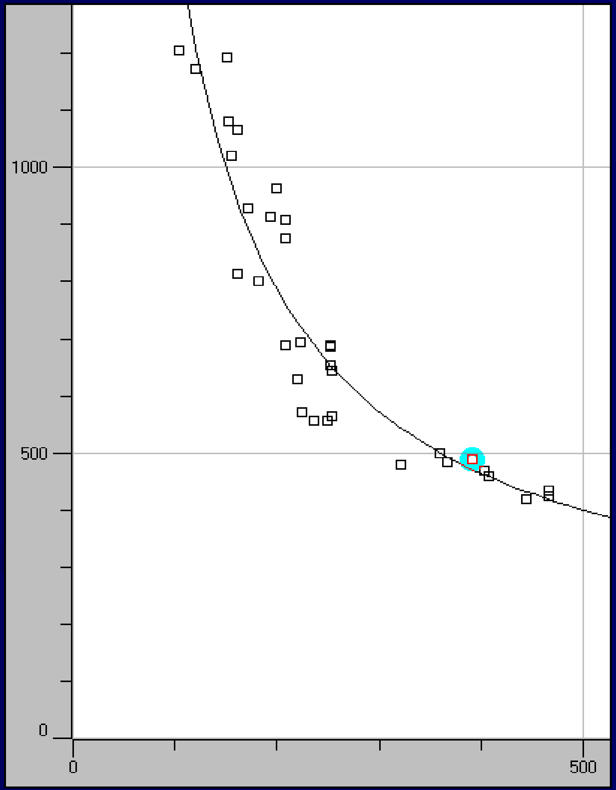
Manual Method:

- A loudspeaker stimulates the saw blade to oscillations
- We touch the edge of the saw on different points

➔ If we touch the vibrating part of the saw, the tone breaks off

- The results of our measurement and a regression function ~ 1/A are ploted in the diagram

Frequency in Hz



Area in cm²

Measurement of the vibrating part of the saw

Laser - Method:

- A loudspeaker stimulates the saw blade to oscillations
- Then a Laser beam is pointed at different parts of the saw blade.
- For the measurements one watches the reflection of the beam on a white wall.

→ If the laser is pointed on a vibrating part of the saw blade the reflection is out of focus.

→ If the laser is pointed on a resting part of the saw blade the reflection is sharp.

Theory:

Equation of motion for transversal waves in thin plates:

$$\left(\frac{\partial^2}{\partial t^2} z(x,y,t)\right) + \frac{\frac{1}{12} E h^2 \left[\left(\frac{\partial^4}{\partial x^4} z(x,y,t)\right) + \left(\frac{\partial^4}{\partial y^4} z(x,y,t)\right) + 2\left(\frac{\partial^4}{\partial y^2 \partial x^2} z(x,y,t)\right)\right]}{\rho (1 - v^2)} = 0 \quad (1)$$

E: Young‘s modul of elasticity
h: Thickness
t: Time
z: Elongation

Formulation for harmonic solutions:

$$z(x, y, t) = Z_o(x, y)\, e^{(j w t)} \qquad (2)$$

z: Elongation of the oscillation

Substitute z(x,y,t) with $Z_o(x,y)\ e^{jwt}$:

$$\left[\left(\frac{\partial^4}{\partial x^4} Z_0(x,y)\right) + \left(\frac{\partial^4}{\partial y^4} Z_0(x,y)\right) + 2\left(\frac{\partial^4}{\partial y^2\, \partial x^2} Z_0(x,y)\right) - \frac{12\, w^2\, Z_0(x,y)}{ct^2\, h^2}\right] = 0 \qquad (3)$$

We define the product:

$$12 \frac{w^2}{c_t^2 h^2} = k^4 \qquad (4)$$

The product k^4 depends on the boundary conditions of the oscillation.

With w = 2πf we get for the frequency f:

$$f = .0459\, k^2\, c_t\, h \qquad (5)$$

We can write $Z_o(x,y)$ as a product of two functions:

$Z_o(x,y) = X(x)*Y(y)$

$$\frac{\frac{\partial^4}{\partial x^4} X(x)}{X(x)} + \frac{\frac{\partial^4}{\partial y^4} Y(y)}{Y(y)} + \frac{2\left(\frac{\partial^2}{\partial x^2} X(x)\right)\left(\frac{\partial^2}{\partial y^2} Y(y)\right)}{X(x)\, Y(y)} = k^4 \qquad (6)$$

Wave equation:

In X- direction:

$$\frac{\frac{\partial^2}{\partial x^2} X(x)}{X(x)} = -k_1^2 \qquad (7)$$

In Y- direction:

$$\frac{\frac{\partial^2}{\partial y^2} Y(y)}{Y(y)} = -k_2^2 \qquad (8)$$

Boundary conditions:

Free edges in x-direction:
Kosine function

$$k_1 = 2\frac{m\pi}{L_x} \qquad (9)$$

Clamped edges in y-direction:
Sinus function

$$k_2 = \frac{n\pi}{L_y} \qquad (10)$$

For the wave number k we get from the differential equation:

$$k_1^2 + k_2^2 = k^2 \qquad (11)$$

With the boundary conditions we get:

$$4\frac{m^2\pi^2}{L_x^2} + \frac{n^2\pi^2}{L_y^2} = k^2 \qquad (12)$$

Put k^2 in $f = .0459\, k^2\, c_t\, h$

Our solution for the frequency is therefore:

$$f = .453\, c_t\, h \left(4 \frac{m^2}{L_x^2} + \frac{n^2}{L_y^2} \right) \qquad (13)$$

m,n: Integers
For our basic oscillation m,n =1
h: Thickness
Lx: Length of the vibrating part
Ly: Width of the vibrating part

Comparison of Experiment and Theory

<u>Example:</u>

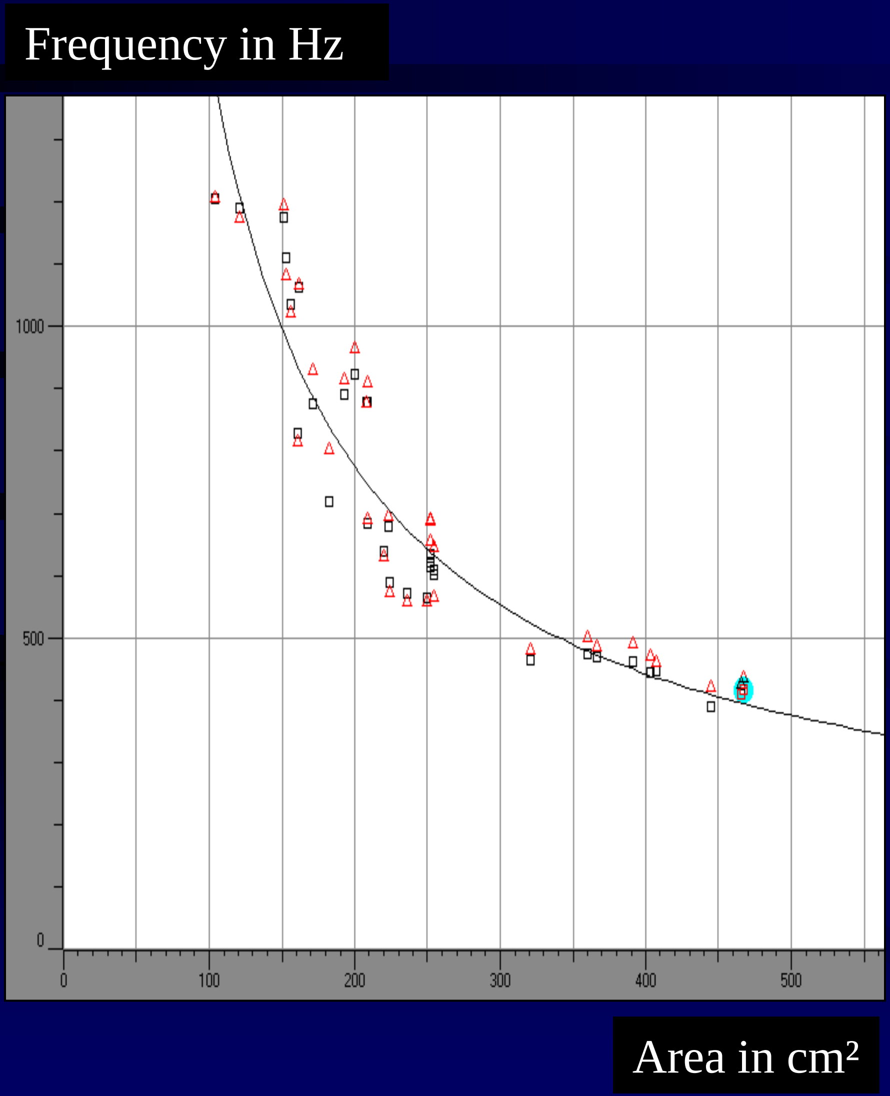
Measured value:	$f = 813$ Hz
Calculated value:	$f = 827$ Hz

With the measured values:

$h = 0.85$ mm $\quad c_t = 3353$ m/s

$L_x = 8.05$ cm $\quad L_y = 20.05$ cm

Comparison of Experiment and Theory



- Calculated values of the frequencies, L_x und L_y are determined experimentally
- Regression function as f ~ 1/A
- Red points: measured values of the frequency by Fourieranalysis

$\Rightarrow$ Good agreement

The frequency depends on the of the vibrating part of the saw

Through transforming of the formula of the frequency we get:

$$f = 0{,}453 c_t t \frac{1}{L_x L_y}\left[\frac{4m^2 L_y}{L_x} + \frac{n^2 L_x}{L_y}\right]$$

➔ The frequency depends on the size of the vibrating part $A = Lx*Ly$ of the saw if Lx/Ly is constant.

➔ We determined Lx/Ly from our measurements: $\alpha = Lx/Ly = 0{,}336 \pm 0{,}042$

Summary:

The frequency depends on:

- The bending of the saw blade which is significant for the size of the vibrating part and the point one bows the saw
- The material of the saw
- The thickness of the saw