BREAKING SPAGHETTI

Yulia Peregud

Belarusian State University Lyceum, Belarus

Formulation of the problem:

Find the conditions under which dry spaghetti falling on a hard floor does not break.

Introduction:

Behavior of dry spaghetti as a brittle rod or behavior of solid bodies during deformations and its mechanical properties was the subject of many researches. But goals of some of these researches ([1], [2], [3], [4], [5]) are investigations of behavior of solid bodies in cases of static deformations. The goal of my research is to describe the processes which occur during dynamic deformation. My research is not the same as the previous researches because: 1) I investigate impact of spaghetti with some surface after free fall of spaghetti (the tips of spaghettis are not fixed). 2) I explain why exactly bending waves are the cause of breaking. 3) I investigate dependences of different important parameters what can change something in our system (but they don't change the cause of breaking).

Dynamic deformation is not the same kind of deformation as static deformation because the causes of breaking of spaghetti during impact with some surface are waves (waves appear in spaghettis during dynamic deformations – [6], [7]). The kind of wave which breaks spaghetti (the kind of wave which dominates in exact case) depends of conditions of an impact of spaghetti with the surface. That's why we must find out what kind of wave occurs in spaghetti to understand the main cause of breaking and, finally find the conditions under which dry spaghetti falling on a floor does not break.

The goal of my research is to determine the most important kind of wave what breaks spaghetti, estimate parameters of this wave, determine the point of maximum curvature and investigate the conditions under what probability of breaking is the least.

Explanation of processes in spaghetti:

To begin with, we should consider three different variants for spaghetti to hit the floor: 1) horizontally, 2) vertically, 3) at some angle between 0° and 90°. The cause of breaking in the first case is compressive strain. But we need enormous amount of energy to compress spaghetti so much. Moreover, it is hard to make such experiments and this kind of impact is not very interesting. The third variant of hitting demands very complicated mathematical model. That's why I investigated only vertical falling of spaghetti.

And now let's explain what exactly happens with spaghetti during the impact. It was mentioned before that the cause of breaking of spaghetti is some kind of wave. There are 4 basic kinds of waves: longitudinal, transverse, torsional, bending waves. Torsional waves don't take place during the impact because spaghetti isn't twisted very much. Transverse waves usually occur in continuous medium, not in rods. Longitudinal waves take place during every impact of solids.

Nevertheless, from the point of view of energy necessary for breaking of spaghetti, the energy necessary for breaking with bending is less than without bending. The influence of longitudinal waves is negligible. Therefore, spaghetti is more likely to break because of curving. Thus, deformation of whole spaghetti is a combination of 2 deformations – some layers of spaghetti are stretched, some are compressed. Using a high-speed camera, I have taken videos of impact. I took some frames (Figure 1) of these videos and in that way proved the theory about bending waves: we can see that spaghetti really bends.

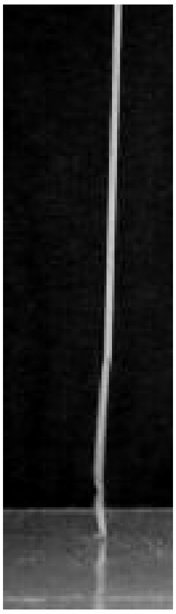

Figure 1: bending of spaghetti at the moment of impact

Now, understanding the nature of processes what occur in spaghetti, we can determine velocity of bending wave ([8]) in spaghetti (c_b)

$$c_b = \sqrt{\frac{2\sqrt{2}\pi \cdot R}{T} \cdot \sqrt{\frac{E}{\rho}}}$$

E – Young modulus of spaghetti, *T* – period of wave, *ρ* – density of spaghetti, *R* – section radius of spaghetti.

I approximated the dependence of reaction force of surface on time as sinusoid and estimated period of wave. The shape of bend of the spaghetti is sinusoid too.

$$T \approx 2t \approx 0{,}004s$$

t – time of the impact, found with the help of high-speed camera (1000 frames/s).

Also I found Young modulus of spaghetti: $\underline{E \approx 2{,}3 \cdot 10^7\ Pa}$.

Now it is possible to estimate the velocity of propagation of bending wave: $c_b \approx 16{,}6$ *m/s.*

As a consequence of these estimations, I can calculate the wave-length (λ):

$$\lambda \approx c_b \cdot T \approx 6cm$$

Bending wave dies out very quickly, that's why not the whole of spaghetti bends. After watching slowed videos of impact I noticed that approximately 3 cm of spaghetti (from the bottom of spaghetti) bend. Further propagation of bending wave does not take place because of energy loss as a result of plastic deformation in spaghetti. Thus, all the energy of bending wave is spent on deformation and, consequently breaking. After breaking there is almost no energy left. Therefore spaghetti usually breaks only 1 time and only in 1 point.

How can we find this point? Spaghetti breaks at the distance ***l ≈ λ/4 ≈ 1,5 cm*** from the bottom because of maximum curvature exactly in this point. The length of broke off pieces of spaghetti was around 1,5 cm in all the experiments. It perfectly proves my theory about bending waves as the main cause of breaking of spaghetti.

Experiment:

First of all, let's clarify the way of characterizing probability of breaking of spaghetti. Let it be *p* – frequency of breaking of spaghetti. *N* – number of spaghetti which broke, N_o – number of all falling spaghetti

$$p = \frac{N}{N_o} \cdot 100\%$$

Secondly, I will describe my experimental setup. It is simple: It is a vertical tube (it's length is 30 cm), fixed in a tripod. Spaghetti falls through this tube without initial velocity.

Thirdly, let's determine the most important factor what has influence on probability of breaking. We can say that this factor is quantity of energy what spaghetti has just before the impact with the surface. If we carry out experiments with the identical spaghettis, this quantity of energy depends only on the height of falling. We have run some experiments, so, you can see the experimental dependence of frequency of breaking on height of falling (figure 2). So, the higher is the kinetic energy of spaghetti, the higher is possibility, that spaghetti bends very much and break.

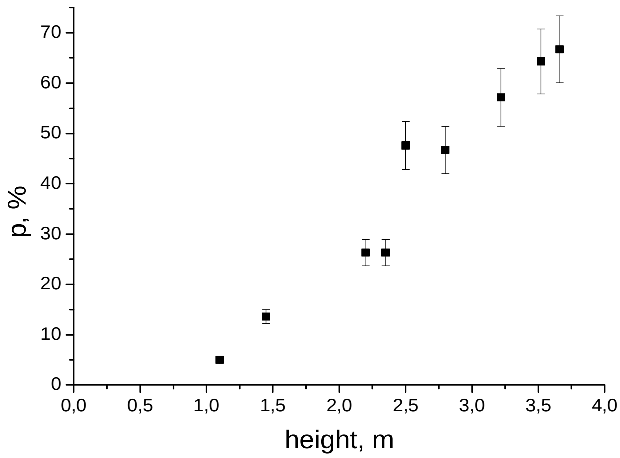

Figure 2: Frequency of breaking vs. height of falling

But there are some other ways of changing kinetic energy: we can change mass of spaghetti by changing length of spaghettis, cutting them (certainly, not changing the diameter). You can see the experimental dependence of frequency of breaking on length of spaghetti (figure 3). We run this experiment keeping the same height of falling. So, we change reaction force of surface by changing kinetic energy of spaghetti

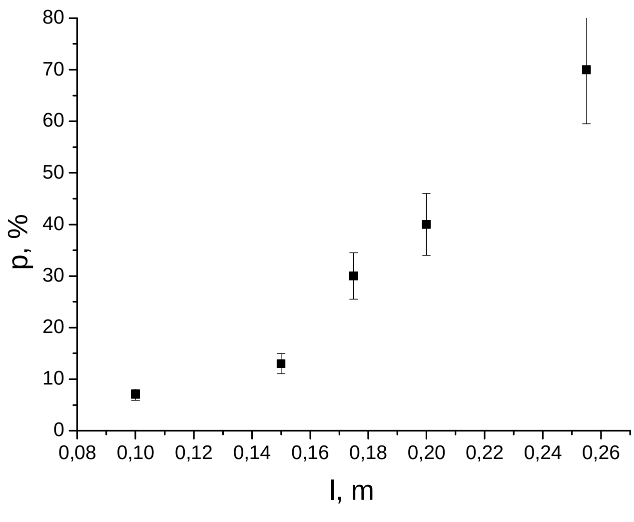

Figure 3: Frequency of breaking vs. length of spaghetti, height = 3,9 m

But also we have the third way of changing kinetic energy of spaghetti. We can run 3 experiments with spaghettis with 3 different diameters and the same length. And what do we see on figure 4 ? Increasing kinetic energy by changing of spaghetti we does not increase frequency of breaking. But what is the reason of such a contradiction? Let's solve this ambiguity: we should notice that when we increase diameter of spaghetti we increase not only mass. Increase diameter it becomes harder to bend spaghetti. It is just hard to believe that we need the same force to bend spaghettis with diameters of 1,45 and 2 mm. It goes without saying that there is some critical curvature for breaking for spaghetti of certain dimensions. Thus, changing of critical curvature is the most important cause of decreasing of frequency of breaking with increasing of diameter of spaghetti. Thus, there can be such a conclusion: it is easier to bend thin spaghettis than not thin one, that's why thinner spaghettis break with higher frequency falling from less height.

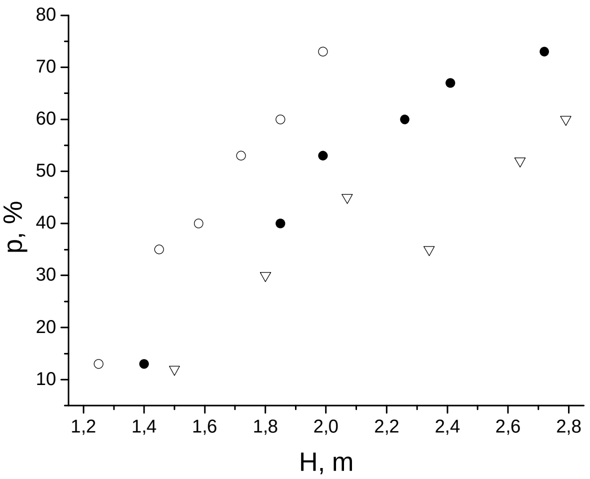

Figure 4: Frequency of breaking vs. height of falling:

empty circles for d=1.4 mm, full – d=1.7 mm, triangles – d=2

There are a lot of different interesting parameters what can have influence on frequency of breaking, but within the limits of IYPT report we cannot investigate all of them. Therefore, we must understand which parameters are the most relevant and which haven't so much physical meaning for us because of too much complicated mathematical formulation of certain parameter. There are 3 the most relevant parameters: 1) height of falling,2) length of spaghetti, 3) diameter of spaghetti. These parameters influence kinetic energy of spaghetti to the utmost: All of these 3 parameters were investigated in my research.

But there is one more parameter what I would like to investigate. It is nature of the surface. I run an experiment with 3 different surfaces (figure 5). It is rather surprising that frequency of breaking is higher when spaghetti hits plastic than granite or metal. It is really hard to explain it numerically because different surfaces have a lot of different properties. Nevertheless, during experiments I noticed that when spaghetti hits metal it jump up very high, when it hits granite – jumps up a bit lower, and when it hits plastic – spaghetti almost doesn't jump. Thus, I can make a conclusion that during the impact with metal great part of kinetic energy lasts out and less quantity of energy spends on deformation. It is the simplest explanation of this surprising effect.

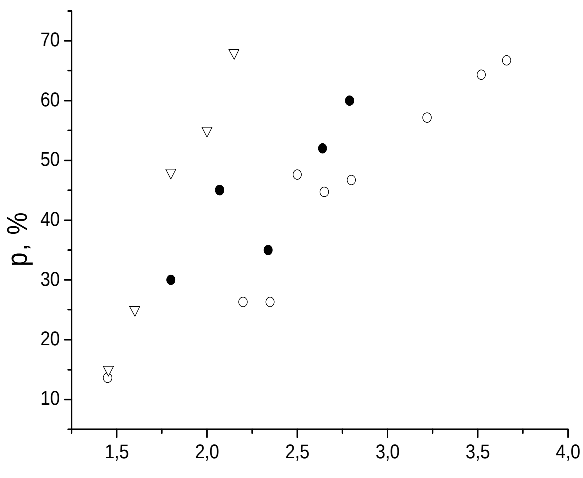

Figure 5: Frequency of breaking vs. height of falling:

empty circles for metal, full – granite, triangles – plastic

Conclusion:

In summary, there is no exact answer of the question "find the conditions under which dry spaghetti falling on a hard floor does not break" because it is really hard to describe influence of all the parameters numerically. We can discuss only frequency of breaking in experimental research and quantity of energy required for breaking of ideal spaghetti in theoretical computation. (The energy needed to break a certain kind of spaghetti is the energy needed to curve spaghetti so much that it will be enough to tear it's fibers. And this curvature will be the critical curvature.) But, summing up all the research, we can suppose that frequency of breaking of an average spaghetti will be low if it falls from the low height (approximately lower than 1,5 m), has big diameter and has not very big length.

Influence of parameters:

There is my summary of influence of parameters in this section:

1) Height: when height of falling increases, frequency of breaking also increases because velocity of spaghetti (consequently, it's kinetic energy) increases when height increases.

2) Length: when length of spaghetti increases, frequency of breaking also increases because mass of spaghetti (consequently, it's kinetic energy) increases when length increases, whereas diameter of spaghetti is the same (consequently, the energy required for breaking of spaghetti is also the same).

3) Diameter: when diameter of spaghetti increases, frequency of breaking also decreases because the energy required for breaking of spaghetti decreases.

4) Surface: when spaghetti hits plastic, frequency of breaking is higher than when spaghetti hits metal. The simplest explanation of it is that during the impact with metal great part of kinetic energy lasts out and less quantity of energy spends on deformation

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